There were editions of this book probably as follows. in London

1	1677 ✓	30	1712 Σ
2		31	
3		32	
4	1682 ✓	33	1715 ✓
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6	1685	35	1718 ✓
7		36	
8		37	1720 ✓
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10		39	1722
11		40	
12		41	
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14	1694	43	1728
15		44	1730
16		45	
17	1697	46	
18	1698	47	1734
19		48	1736
20	1700 ✓	49	
21		50	1741
22		51	
23		52	1749
24	1706	53	1750
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26	1708 Σ	55	1758
27		56	1767 Σ
28		57	1769
29	1711	58	
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A
TREATISE
OF
ARITHMETIC,

BEING

A plai[illegible] familiar method, suitable to the mea[illegible] capacity, for the full understanding of that incomparable Art.

BY

EDWARD COCKER.

A NEW EDITION,

Revised and corrected by JOHN MAIR, A. M.

EDINBURGH:

Printed by A. DONALDSON and J. REID.
For E. WILSON Bookseller in *Dumfries*.
MDCCLXV.

[Price One Shilling bound.]

COCKER's VULGAR ARITHMETIC, though in high eſteem for many years after its firſt publication, and though a book ſtill much called for, has nevertheleſs had the misfortune, for a long time paſt, to have been very careleſsly printed; all the modern impreſſions, without exception, ſo much abounding in omiſſions, falſe figures, erroneous anſwers, and typographical blunders of every ſort, that the book, in many places, tends to miſlead or puzzle, rather than inſtruct the learner. And it being the deſign of the preſent editor to remedy this evil, I have, in compliance with his repeated intreaties, and taking for my guide the moſt ancient editions, particularly that publiſhed at London 1697, endeavoured to ſupply defects, and rectify miſtakes of every kind, in order to reſtore the book to its primitive purity, and once more furniſh the public with a genuine and correct copy of this part of Mr COCKER's works:——That it may anſwer the end propoſed by the editor, and prove ſerviceable to the reader, is the earneſt deſire of him who wiſheth the propagation of every uſeful art and ſcience.

Auguſt 1. 1751.

JOHN MAIR.

To his Much Honoured FRIENDS,

MANWARING DAVIES, of the INNER TEMPLE, Esq;

AND

Mr HUMPHREY DAVIES, of St MARY, NEWINGTON BUTTS, in the county of SURRY,

JOHN HAWKINS, as an acknowledgment of unmerited favours, humbly dedicateth this MANUAL OF ARITHMETIC.

To the READER.

Courteous Reader,

I Having had the happiness of an intimate acquaintance with Mr Cocker in his lifetime, often solicited him to remember his promise to the world, of publishing his arithmetic; but (for reasons best known to himself) he refused it; and (after his death) the copy falling accidentally into my hands, I thought it not convenient to smother a work of so considerable a moment; not questioning but it might be as kindly accepted, as if it had been presented by his own hand. The method is familiar and easy, discovering as well the theoric as the practic of that necessary art of vulgar arithmetic; and in this new edition there are many remarkable alterations, for the benefit of the teacher or learner, which I hope will be very acceptable to the world. I have also performed my promise in publishing the decimal arithmetic, which finds encouragement to my expectation and the booksellers too.——I am,

Thine to serve thee,

JOHN HAWKINS.

Courteous Reader,

BEing well acquainted with the deceased author, and finding him knowing and studious in the mysteries of numbers and algebra, of which he had some choice manuscripts, and a great collection of printed authors in several languages, I doubt not but he hath writ his arithmetic suitable to his own preface, and worthy acceptation; which I thought fit to certify on a request to that purpose made to him that wisheth thy welfare, and the progress of arts.

November 27.
1677.

JOHN COLLINS.

This manual of arithmetic is recommended to the world by us whose names are subscribed, viz.

Mr	*Mr*	*Mr*
Jo. Collens, } Mat.	*Rich. Noble of Guilford.*	*Benj. Tichbourn*
Ja. Atkinson, } Mat.	*William Norgate*	*Joseph Symmonds*
Peter Perkins, } Mat.	*William Mason*	*Jonem. Milles*
Rich. Lawrence, sen.	*Steph. Thomas*	*Josiah Cuffly*
Eleazar Wigan	*Peter Storey*	*John Hawkins*

And generally approved by all ingenious artists.

THE AUTHOR'S

PREFACE.

BY the sacred influence of divine providence, I have been instrumental to the benefit of many, by virtue of those useful arts, writing and engraving: and do now, with the same wonted alacrity, cast this my arithmetical mite into the public treasury, beseeching the Almighty to grant the like blessing to these as to my former labours.

Seven sciences, supremely excellent,
Are the chief stars in wisdom's firmament;
Whereof ARITHMETIC is one, whose worth
The beams of profit and delight shine forth:
This crowns the rest, this makes man's mind complete;
This treats of numbers, and of this we treat.

I have been often desired by my intimate friends to publish something on this subject; who in a pleasing freedom have signified to me, that they expected it would be extraordinary. How far I have answered their expectation, I know not: but this I know, that I have designed this work not extraordinary abstruse or profound; but have, by all means possible within the circumference of my capacity, endeavoured to render it extraordinary useful to all those whose occasions shall induce them to make use of numbers. If it be objected, That the books already published, treating of numbers, are innumerable; I answer, That it is but a small wonder, since the art is infinite. But that there should be so many excellent tracts of practical arithmetic extant, and so little practised, is to me a greater wonder; knowing, that as merchandise is the life of the weal-public, so practical arithmetic is the soul of merchandise. Therefore I do ingenuously profess, that, in the beginning of this undertaking, the numerous concerns of the honoured merchant first possessed my consideration:

and

and how far I have accommodated this composure for his most worthy service, let his own profitable experience be judge.

Secondly, For your service, most excellent professors, whose understandings soar to the sublimity of the theory and practice of this noble science, was this arithmetical tractate composed; which you may please to employ as a monitor to instruct your young tyroes, and thereby take occasion to reserve your precious moments, which might be exhausted that way, for your more important affairs.

Thirdly, For you the ingenious offspring of happy parents, who will willingly pay the full price of industry and exercise for those arts and choice accomplishments, which may contribute to the felicity of your future state; for you, I say, ingenious practitioners, was this work composed, which may prove the pleasure of your youth, and the glory of your age.

Lastly, For you the pretended numerists of this vapouring age, who are more disingeniously witty to propound unnecessary questions, than ingeniously judicious to resolve such as are necessary; for you was this book composed and published: if you will deny yourselves so much as not to invert the streams of your ingenuity, but, by studiously conferring with the notes, names, orders, progress, species, properties, proprieties, proportions, powers, affections, and applications of numbers delivered herein, become such artists indeed, as you now only seem to be. This arithmetic, ingeniously observed, and diligently practised, will turn to good account to all that shall be concerned in accounts; all whose rules are grounded on verity, and delivered with sincerity. The examples are built up gradually from the smallest consideration to the greatest. All the problems or propositions are well weighed, pertinent, and clear; and not one of them throughout the tract taken upon trust. Therefore now,

Zoilus and Momus, lie you down and die;
For these inventions your whole force defy.

EDWARD COCKER.

A TABLE of the CONTENTS.

ARITHMETIC.

CHAP. I.

Notation of Numbers.

ARITHMETIC is the art of numbering, or knowledge which teacheth to number well, *viz.* the doctrine of accounting by numbers. And there are divers species and kinds of arithmetic and geometry; the which we do intend to treat of in order; applying the principles of the one to the definitions of the other. For as magnitude or greatness is the subject of geometry, so multitude or number is the subject of arithmetic; and if so, then their first principles and chief fundamentals must have like definitions, or at least some congruency.

2. Number is that by which the quantity of any thing is expressed or numbered: as the unit is the number by which the quantity of one thing is expressed, or said to be *one;* and two by which it is named *two;* and $\frac{1}{2}$, by which it is named or called *half;* and the root of 3, by which it is called *the root of* 3; the like of any other.

3. Hence it is that unit is number. For the part is of the same matter that is its whole; the unit is part of the multitude of units, therefore the unit is of the same matter that is the multitude of units: but the matter of the multitude of units is number, therefore the matter of unit is number. For else, if from a number given no number be subtracted, the number given remaineth. Let three be the number given; from which number subtract or take away one, (which, as some conceive, is no number), therefore the number given remaineth; that is to say, there remaineth three: which is absurd.

4. Hence it will be convenient to examine from whence number hath its rise or beginning. Most authors maintain, that unit is the beginning of number, and itself no number: but looking upon the principles and definitions in

in the first rudiments of geometry, we shall find that the definition of a point is no way congruous with the definition of an unit in arithmetic; and therefore one or unit must be in the bounds or limits of number, and consequently the beginning of number is not to be found in the number one. Wherefore, when we make number and magnitude congruent in principles, and like in definitions, we make and constitute a cipher to be the beginning of number, or rather the medium between increasing and decreasing numbers, commonly called *absolute* or *whole numbers*, and *negative* or *fractional numbers*. Than which nothing can be imagined more agreeable to the definition of a point in geometry. For as a point is an adjunct of a line, and itself no line; so is a (o) cipher an adjunct of number, and itself no number: and as a point in geometry cannot be divided or increased into parts; so likewise (o) cannot be divided or increased into parts. For as many points, though in number infinite, do make no line; so many (o) ciphers, though in number infinite, do make no number. For the line A B cannot be increased by the addition of the point C, neither can the number D be increased by the addition of the (o) cipher E; for if you add nothing to 6, the sum will be 6, the (o) cipher neither increasing nor diminishing the number 6: but if it be granted, that A B be extended or prolonged to the point C, so that A C be made a continued line; then A B is increased by the addition of the point C. In like manner, if we grant D (6) be prolonged to E (o), so that D E (60) be a continued number, making 60; then 6 is augmented by the aid of (o), as constituting the number (60) sixty. And furthermore, that one or unit is material, and a number, and that (o) is the beginning of number, is proved by all authors, although indirectly: for the tables of sines and tangents prove one degree to be a number; because the sine of 1 degree is 174524, (the radius being 10000000), and the beginning of that table is (o); and to it answereth 00000, &c.

A————·————B
C

D	6
E	o
Sum	6

A————B————C

$\left.\begin{matrix} D\ E \\ 6\ o \end{matrix}\right\}$ 60

5. Hence

5. Hence it is that number is not quantity discontinued. For that which is but one quantity, is not quantity disjunct: (60) sixty, as it is a number, is one quantity, *viz.* one number (60) sixty; therefore, as it is number, it is not quantity disjunct. For number is some such thing in magnitude, as humidity in water; for as humidity extends itself through all and every part of water, so number related to magnitude, doth extend itself through all and every part of magnitude. Also, as to continued water doth answer continued humidity, so to a continued magnitude doth answer a continued number: as the continued humidity of water suffereth the same division and distinction that the water doth, so continued number suffereth the same division and distinction that its magnitude doth. From all which considerations we might enlarge farther concerning number and magnitude, by comparing the definitions of the one with the principles of the other: for having found a (0) cipher to answer in definition to a point in magnitude, we may very well conclude, that number may be congruent to a line; as also a figurative number to be consonant in definition with a superficies and solid, *&c.* in the order of geometrical magnitudes.

6. The characters or notes by which numbers are signified, or by which a number is ordinarily expressed, are these following, *viz.* 0, cipher, or nothing; 1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine. The cipher, which though of itself it signifieth nothing, *viz.* expresseth not any certain or known quantity; yet it is the beginning, radix, or root of number; and the other nine characters are called *significant figures*, or *digits*.

7. In numbers of any sort, two things are to be considered, *viz.* notation and numeration.

8. Notation teacheth how to describe any number by certain notes and characters, and to declare the value thereof being so described; and that is by degrees and periods.

9. A degree consists of three figures, *viz.* of three places, comprehending units, tens, and hundreds. So 365 is a degree: and the first figure (5) on the right hand stands simply for its own value, being units, or so many ones,

ones, *viz.* five; the ſecond in order from the right ſignifies as many times ten as there are units contained in it, *viz.* ſixty; the third in the ſame order ſignifies ſo many hundreds as it contains units; ſo will the expreſſion of the number be three hundred ſixty-five; alſo 789, is ſeven hundred eighty-nine, &c.

10. A period is, when a number conſiſts of more than three figures or places, according to whoſe proper order we are to prick or diſtinguiſh every third place, beginning at the right hand, and ſo on to the left. So the number 63452 being given, it will be diſtinguiſhed thus, 63,452, and expreſſed thus, ſixty-three thouſand, four hundred fifty-two. Likewiſe 4,578,236,782 being diſtinguiſhed as you ſee, will be expreſſed thus, four thouſand five hundred ſeventy-eight millions, two hundred thirty-ſix thouſand, ſeven hundred eighty-two.

11. Number is either abſolute or negative.

12. An abſolute, or entire, whole, increaſing number, is that which, by annexing another figure or cipher, becomes ten times ſo much as it ſtood for before; and if two figures or ciphers be annexed, it makes an hundred times as much as it ſtood for before, &c.: As, if you annex to the figure 6 a cipher, then it will become 60, ſixty; ſo if two ciphers are annexed, then it will be (600) ſix hundred; and if you do annex to it a (4) four, then it will be (64) ſixty-four; and if you annex (78) ſeventy-eight, it will be then (678) ſix hundred ſeventy-eight; and ſo on. By annexing more figures or ciphers, it will increaſe in a decuple proportion *ad infinitum*.

13. A negative or broken, fractional, decreaſing number, is that which, by prefixing a point or prick towards the left hand, its value is decreaſed from ſo many units, to ſo many tenth parts of any thing; and if a point and (0) cipher or digit be prefixed, it will be then ſo many hundredth parts; and if a point and two ciphers or digits be prefixed, its value is decreaſed to be ſo many thouſandth parts: As if you would prefix before the figure 3 a point (.) or prick thus (.3), it is then decreaſed from 3 units or 3 integers, to three tenth parts of an unit or integer; and if you prefix a point and cipher thus (.03), it is decreaſed from 3 integers to three hundred parts of an integer:

integer: and by this means 5 *l.* abſolute, by prefixing of a point, will be decreaſed to 5 *l.* negative; which is 5 tenth parts of a pound, equal in value to ten ſhillings; and ſo by prefixing of more ciphers or digits, its value is decreaſed in a decuple proportion *ad infinitum*: As in the following ſcheme, or order of numbers, we have placed (0) cipher in its due place and order, as it is the beginning and medium of number; for going from (0) towards the left hand, you deal with entire, abſolute, whole, increaſing numbers.

Increaſing Numbers. *Decreaſing Numbers.*

876	543	256	2 1 0 1 2	345	678	976
			C.X.U.X.C.			
C.X. Thouſ. of millions.	C.X. Millions.	C.X. Thouſands.	Hundreds. Tens. Units. Tenth parts. Hund. parts.	Thouſ. parts. X.C.	Million parts. X.C.	Th. mill. parts. X.C.

But going from (0) the place of units, towards the right hand, you meet with broken, negative, fractional, and decreaſing numbers. And hence it follows, that multiplication increaſeth the product in abſolute numbers, but decreaſeth the product in negative numbers. Alſo diviſion decreaſeth the quotient in whole numbers, and increaſeth it in negative or fractional numbers.

14. An abſolute, entire, whole, increaſing number, hath always a point annexed towards the right hand; and therefore,

15. A negative, broken, decimal, decreaſing number, hath always a point prefixed before it towards the left hand. When we expreſs integers or whole numbers, as 5 pounds, 5 feet, 26 men, we uſually annex a

l. feet. men. inch.

point or prick after the number thus, 5. 5. 26. 347. But when we expreſs decimals, or decreaſing numbers, we do commonly prefix a point or prick before the ſaid decimal, or decreaſing number, thus (.3), that is three tenths, or 3 primes; (.03), that is, 3 hundredths, or 3 ſeconds.

16. A

16. A whole or absolute number is an unit, or a composed multitude of units; and it is either a prime, or else a composite number.

17. Prime numbers amongst themselves, are those which have no multitude of units for a common measurer, as 8 and 7, or 10 and 13; because not any multitude of units can equally measure or divide them without a remainder.

18. Composite numbers amongst themselves, are those which have a multitude of units for a common measurer, as 9 and 12; because three measures them exactly, and abbreviates them to 3 and 4.

19. A broken number, commonly called a *fraction*, is a part or parts of a whole number, *viz.* a part of an integer; as ($\frac{1}{3}$), one third, is one third part of an unit.

20. A broken number or fraction consists of two parts, *viz.* the numerator and denominator.

21. The numerator and denominator of a fraction are set one over the other, with a line between them; and the numerator is set above the line, and expresseth the number of parts therein contained.

22. The denominator of a fraction, is the inferior number placed below the line, and expresseth the number of parts into which the unit or integer is divided: As, let $\frac{3}{4}$ be the fraction given, so shall 3 be the numerator, and doth express or number the multitude of parts contained in this fraction; for $\frac{3}{4}$ is a fraction compounded of fourths, or quarters, and the figure 3 in numbering, shews us, that in that fraction there are three of those fourth parts or quarters: also in the same fraction $\frac{3}{4}$, 4 is the denominator, and doth express the quality of the fraction, *viz.* that the whole or integer is divided into four equal parts.

23. A broken number is either proper or improper, *viz.* proper, when the numerator is less than tha denomidator; thus $\frac{1}{2}$ is a proper fraction: but an improper fraction hath its numerator greater, or at least equal to the denominator; thus $\frac{13}{8}$ and $\frac{8}{8}$ are improper fractions.

24. A proper broken number is either simple or compound, *viz.* simple, when it hath one denominator, and compound, when it consisteth of divers denominators. If $\frac{3}{4}$, $\frac{9}{11}$, $\frac{15}{120}$, were given, we say, they are each of them

single

ſingle or ſimple fractions, becauſe they conſiſt but of one numerator, and one denominator; but if $\frac{3}{4}$ of $\frac{9}{12}$ of $\frac{15}{120}$ of a pound Sterling were given, we ſay that it is a compound broken number or fraction, becauſe the expreſſion and repreſentation conſiſteth of more denominators than one. And ſuch by ſome are called *fractions of fractions*. They have always the particle *of* between them.

25. When a ſingle broken number or fraction hath for its denominator a number conſiſting of an unit in the firſt place toward the left hand, and nothing but ciphers from the unit toward the right hand, it is then aptly and rightly called a *decimal fraction*. Under this head are all our decreaſing numbers placed, and in our 13th definition called *negatives*. And by the order there preſcribed, we ſignify them to be decimals, by putting a point or prick before them, or before the numerator, rejecting the denominator. Therefore, according to our laſt rule, $\frac{5}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$, are ſaid to be decimals: and a decimal fraction may be expreſſed without its denominator, as before, by prefixing a point or prick before the numerator of the ſaid fraction, and then ſhall the former fractions $\frac{5}{10}$, $\frac{5}{100}$, and $\frac{25}{1000}$ ſtand thus, .5, .05, and .025.

But oftentimes, as in the ſecond and third fraction, $\frac{5}{100}$ and $\frac{25}{1000}$, a prick or point will not do without the help of a cipher or ciphers prefixed before the ſignificant figures of the numerator: and therefore when the numerator of a decimal fraction conſiſteth not of ſo many places as the denominator hath ciphers, fill up the void places of the numerator, by prefixing ciphers before the ſignificant figures of the numerator, and then ſign it for a decimal; ſo ſhall $\frac{5}{100}$ be .05, and $\frac{25}{1000}$ will be .025, and $\frac{72}{10000}$ will be .0072. Now, by this we may eaſily diſcover the denominator, having the numerator: for always the denominator of any decimal fraction conſiſts of ſo many ciphers as the numerator hath places, with an unit prefixed before the ſaid cipher, *viz.* under the point or prick.

26. A decimal number or fraction, is expreſſed by *primes*, *ſeconds*, *thirds*, *fourths*, &c.; and is number decreaſing. Here, inſtead of natural and common fractions, as $\frac{3}{4}$ of a thing, we diſtribute the thing or integer into

primes, *seconds*, *thirds*, *fourths*, *fifths*, &c. that our expression may be consonant to our former order.

27. In decimal arithmetic we always imagine, and it would be very commodious if it were really so, that all entire units, integers, and things, are divided, first, into ten equal parts; and these parts so divided we call *primes*: and, secondly, we divide also each of the former primes into other ten equal parts; and every one of these divisions we call *seconds*: and thirdly, we divide each of the said seconds into ten other equal parts; and those so divided we call *thirds*: and so by decimating the former, and subdecimating the latter, we run on *ad infinitum*.

28. Let a pound Sterling, Troy weight, Avoirdupois weight, liquid measure, dry measure, long measure, time, dozen, or any other thing, or integer, be given to be decimally divided; according to the notion premised, we ought to let the first division be primes, the next division seconds, the next thirds, *&c.* So one pound Sterling being 20 shillings, when divided into ten equal parts, the value of each part will be 2 shillings; therefore one prime of a pound Sterling will stand thus, (.1), which is in value 2 shillings; three primes will stand thus (.3), and that is in value 6 shillings. Again, a prime, or .1, being devided into ten equal parts, each of those parts will be one second, and is thus expressed, (.01); and its value will be found to be 2 d. farthing and $\frac{6}{10}$ of a farthing; and so will .05 signify one shilling, or five seconds. And if .01 be divided into ten other equal parts, each of those parts so divided will be thirds, and will stand thus, .001; and its value will be found to be .96 of a farthing, or $\frac{96}{100}$ of a farthing; and .009 thirds will be 2 d. and .64 of a farthing, or $\frac{64}{100}$ of a farthing, *&c.* So that .375 l. will be found to represent 7 s. 6 d.; for the three primes are 6 shillings, and the seven seconds are 1 s. 4 d. and $\frac{8}{10}$ of a penny, and the five thirds are 1 penny and $\frac{2}{10}$ of a penny; which, added together, make 7 s. 6 d.

29. If you put any bulk or body representing an integer, if it be decimally divided, then the parts in the first decimation are primes, the next seconds, and the next decimation is thirds, the next fourths, *&c.* As, let there be given a bullet of lead, or such like, whose weight let be

be 50 lb. Troy; this is called an *unit, integer*, or *thing*; then will the like weight and matter make 10 other, the which together, will be equal to 50 lb. and will weigh each of them 5 lb. apiece. Take of the same matter, equal to 5 lb. and make 10 more, then each of those will weigh 6 ounces apiece. Also, if again you take 6 ounces, and thereof make 10 other small bullets, each of them will weigh 12 penny-weight Troy. And thus have you made primes, seconds, and thirds, in respect of the integer, containing 50 lb. Troy weight; so that five primes are equal to the half mass, and two primes and five seconds is a quarter of the mass; and therefore one of the first division, 2 of the second division, and 5 of the third division, will be equal in weight to half a quarter of the mass, and will contain 6 lb. 3 oz.

30. When a decimal fraction followeth a whole number, you are to separate or part the decimal from the whole number by a point or prick. So if .75 followed the whole number 32, set them thus, 32.75. You shall find that divers authors have divers ways in expressing mixed numbers; as thus, 32|75, or $32\frac{75}{100}$, or 3275; but you will find that 32.75, thus placed and expressed, is the fittest for calculation.

31. A mixed number hath two parts; the whole, and the broken. The whole is that which is composed of integers; and the broken is a fraction annexed thereunto. So the mixed number $36\frac{8}{12}$ being given, we say, that 36 is the whole number, which is composed of integers, and the $\frac{8}{12}$ is the broken number annexed; which sheweth, that one of the former integers, (*viz.* one of 36), being divided into 12 parts, $\frac{8}{12}$ doth express 8 of those 12 parts more belonging to the said 36 integers.

32. Denominative numbers are of one or of many. And those are of divers sorts and kinds, *viz.* singular, called *unity*, as 1; and plural, called *multitude*, as 2, 3, 4, 5: single, of one kind only, called *digits*, as 1, 2, 3, 4, 5, 6, 7, 8, 9; and compounds of many, as 10, 11, 12, &c. 102, 367, &c.:

Proportional, as single, multiple, double, triple, quadruple, &c.: denominate, as pounds, shillings, pence; undenominate,

denominate, as 1, 2, 3, *&c.*: perfect, as 6, 28, 496, 8128, 130816, 2096128, *&c.* whose parts are equal to the numbers; imperfect, unequal, and more than the sum, as 12, to 1, 2, 3, 4, 6; imperfect, unequal, and less than the sum, as 8, to 1, 2, 4: numbers commensurable and incommensurable, as 12 and 9 are commensurable, because 3 measures them both; but 6 and 17 are incommensurable, because no one common number or measure can measure them: lineal, in form of a line, as ....; superficial, in form of a superficies or plane, as :::::, or ⋮ ⋮, *&c.* and number cubical or solid in form of a cube. These two latter are otherwise called *figurative numbers.* There are also other numbers called *tabular*, as sines, tangents, secants, *&c.*: others that are called *logarithmetic*, or *borrowed numbers*, fitted to proportion for ease and speedy calculation of all manner of questions.

C H A P. II.

Of the natural division of integers, and the several denominations of the parts.

1. BEfore we come to calculation, or the ordering of numbers to operate any arithmetical question proposed, we will lay down tables of the denomination of several integers; and after that (having mentioned the several species and kinds of arithmetic) we shall immediately handle the species of numeration, which are the main pillars upon which the whole fabric of this art is built.

Of money, weights, &c.

2. The least denomination or fraction of money used in England is a farthing, from whence is produced the following table, called *the table of coin*, viz.

And therefore,

				l.	*s.*	*d.*	*qrs.*
1 Farth.	make	1 Farthing		1	20	12	4
4 Farth.		1 Penny					
12 Pence		1 Shilling		1	20	240	960
20 Shill.		1 Pound			1	12	48
						1	4

The

The first of these tables, *viz.* that on the left hand, is plain and easy to be understood, and therefore wants no directions. In the second table, above the line you have 1 l. 20 s. 12 d. 4 qrs.; whereby is meant, that 1 pound is equal to 20 shillings, and 1 shilling is equal to 12 pence, and 1 penny is equal to 4 farthings. Under the line is 1 l. 20 s. 240 d. 960 qrs.; which signifies 1 pound to contain 20 shillings, or 240 pence, or 960 farthings. In the second line below that is 1 s. 12 d. 48 qrs. the first standing under the denomination of shillings; whereby is noted, that 1 shilling is equal to 12 pence, or 48 farthings. And likewise in the line below that, one penny is equal in value to four farthings. Understand the like reason in all the following tables of weight and measure, time, motion, and dozen.

Of Troy weight.

3. The least fraction or denomination of weight used in England, is a grain of wheat gathered out of the middle of the ear, and well dried; from whence are produced these following tables of weight called *Troy weight.*

32 Grains of wheat	make	24 Artificial grains
24 Artificial grains	make	1 Penny-weight
20 Penny weight	make	1 Ounce
12 Ounces	make	1 Pound Troy weight

And therefore,

lb.	*oz.*	*pw.*	*grains.*
1	12	20	24
1	12	240	5760
	1	20	480
		1	24

Troy weight serveth to weigh bread, gold, silver, and electuaries. It also regulateth and prescribeth a form how to keep the money of England at a certain standard. The goldsmiths have divided the ounce Troy weight into other parts, which they generally call *mark weight.* The denominative parts thereof are as followeth, *viz.* a mark (being an ounce Troy) is divided into 24 equal parts, called *carects*, and each carect into 4 grains; so that in a

mark

mark are 96 grains. By this weight they diſtinguiſh the different fineneſs of their gold; for if to 22 carects of gold be put 2 carects of alloy, (which is of ſilver, copper, or other baſer metal, with which they uſe to mix their gold or ſilver to abate the fineneſs thereof), both making when mixed but an ounce or 24 carects, then this gold is ſaid to be 22 carects fine; for if it come to be refined, the 2 carects of alloy will fly away, and leave only 22 carects of pure gold: the like to be conſidered of a greater or leſſer quantity. And as the fineneſs of gold is eſtimated by carects, ſo the fineneſs of ſilver is diſtinguiſhed by ounces; for if a pound of it be pure, and loſeth nothing in the refining, ſuch ſilver is ſaid to be twelve ounces fine; but if it loſeth any thing, it is ſaid to contain ſo much fineneſs as the loſs wanteth of twelve ounces; as if it loſt 1 ounce 14 penny-weight, then it is ſaid to be 10 ounces 6 penny weight fine; and that which loſeth 2 ounces 4 penny-weight 16 grains, is ſaid to be 9 ounces 15 penny-weight 8 grains fine, *&c.*; the like of a greater or leſſer quantity.

Of apothecaries weight.

4. The apothecaries have their weights deduced from Troy weight, a pound Troy being the greateſt integer; a table of whoſe diviſion and ſubdiviſion followeth, *viz.*

			And therefore,				
			lb.	*oz.*	*dr.*	*ſcr.*	*gr.*
1 Pound	makes	12 Ounces	1	12	8	3	20
1 Ounce		8 Drams					
1 Dram		3 Scruples	1	12	96	288	5760
1 Scruple		20 Grains		1	8	24	480
					1	3	60
						1	20

5. Thus much concerning Troy weight, and its derivative weights; which, as was ſaid before, ſerveth to weigh bread, gold, ſilver, and electuaries. Now, beſides Troy weight, there is another kind of weight uſed in England, commonly known by the name of *Avoirdupois weight*, (1 pound of which is equal to 14 ounces 12 penny-weight Troy weight); and it ſerveth to weigh all

all kind of grocery wares, as alſo butter, cheeſe, fleſh, wax, tallow, roſin, pitch, lead, and all ſuch kind of garble; the table of which weight is as followeth.

The table of Avoirdupois weight.

4 Quarters of a dram	make	1 Dram
16 Drams		1 Ounce
16 Ounces		1 Pound
28 Pounds		1 Quarter of a hundred
4 Quarters		1 Hund^d weight or 112 lb.
20 Hundred		1 Tun

And therefore,

Tun.	*C.*	*qrs.*	*lb.*	*oz.*	*dr.*	*qrs.*
1	20	4	28	16	16	4
1	20	80	2240	35840	573440	2293760
	1	4	112	1792	28672	114688
		1	28	448	7168	28672
			1	16	256	1024
				1	16	64
					1	4

Wool is weighed with this weight, but only the diviſions are not the ſame; a table whereof followeth.

A table of the denominative parts of wool-weight.

7 Pound	make	1 Clove
2 Cloves		1 Stone
2 Stones		1 Todd
6 Todd 1 ſtone		1 Wey
2 Weys		1 Sack
12 Sacks		1 Laſt

And therefore,

Laſt.	*ſack.*	*wey.*	*todd.*	*ſtone.*	*cloves.*	*lb.*
1	12	2	6½	2	2	7
1	12	24	156	312	624	44368
	1	2	13	26	52	364
		1	6½	13	26	182
			1	2	4	28
				1	2	14
					1	7

Note,

Note, That in some counties the wey is 256 lb. Avoirdupois, as in the Suffolk wey; but in Essex there is 336 lb. in a wey.

6. The least denominative part of liquid measure is a pint, which was formerly taken from Troy weight, (1 lb. of wheat Troy weight making a pint of liquid measure): but in regard of the difference between the brewers and farmers of his Majesty's excise, concerning the gauging of vessels, occasioned by the different opinions of artists concerning the solid inches in a gallon, it was lately decided by act of parliament; the statute now making 282 solid inches in a beer gallon, and 231 in a wine gallon; and consequently the pint beer measure to contain $35\frac{1}{4}$ solid inches, and the pint wine measure to contain $28\frac{7}{8}$ cubical or solid inches. From whence is drawn the following table.

The table of liquid measure.

$35\frac{1}{4}$ Cubical inches	make	1 Pint beer measure
$28\frac{7}{8}$ Cubical inches		1 Pint wine measure
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of ale, soap, or herring
9 Gallons		1 Firkin of beer
10 Gallons and a half		1 Firk. of salmon or eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of wine
63 Gallons		1 Hogshead
2 Hogsheads		1 Pipe or butt
2 Pipes or butts		1 Tune of wine

And therefore,

Tuns.	*pipes.*	*hhds.*	*gall.*	*pints*
1	2	2	63	8
1	2	4	252	2016
	1	2	126	1008
		1	63	504
			1	8

7. The

7. The leaſt denominative part of dry meaſure is alſo a pint, and this is likewiſe taken from Troy weight: the table of whoſe diviſion followeth.

The table of dry meaſure.

1 Pound Troy	make	1 Pint
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Buſhel
4 Buſhels		1 Comb
2 Combs		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey
2 Weys		1 Laſt

And therefore,

Laſt.	*wey.*	*qrs.*	*com.*	*buſh.*	*pecks.*	*gall.*	*pints.*
1	2	5	2	4	4	2	8
1	2	10	20	80	320	640	5120
	1	5	10	40	160	320	2560
		1	2	8	32	64	512
			1	4	16	32	256
				1	4	8	64
					1	2	16
						1	8

8. The leaſt denominative part of long meaſure is a barley-corn well dried, and taken out of the middle of the ear; whoſe table of parts followeth.

3 Barley-corns	make	1 Inch
12 Inches		1 Foot
3 Feet		1 Yard
3 Feet 9 inches, or a yard and a quarter		1 Ell Engliſh
6 Feet		1 Fathom
5 Yards and a half		1 Pole, perch, or rod
40 Poles or perches		1 Furlong
8 Furlongs		1 Engliſh mile

And

And therefore,

Mile.	*furl.*	*poles.*	*yards.*	*feet.*	*inches.*	*barley-corns.*
1	8	40	$5\frac{1}{2}$	3	12	3
1	8	320	1760	5280	63360	190080
	1	40	220	660	7920	23760
		1	$5\frac{1}{2}$	$16\frac{1}{2}$	198	594
			1	3	36	108
				1	12	36
					1	3

And note, That the yard, as also the ell, is usually divided into 4 quarters, and each quarter into 4 nails.

Note also, That a geometrical pace is 5 feet; and there are 1056 such paces in an English mile.

9. The parts of the superficial measures of land, are such as are mentioned in the following table, *viz.*

A table of land-measure.

40 Square poles or perches	make	1 Rood or quarter of an acre
4 Roods	make	1 Acre

By the foregoing table of long measure, you are informed what a pole, or, which is all one, a perch, is; and by this, that 40 square perches is a rood. Now, a square perch is a superficies very aptly resembled by a square trencher, every side thereof being a perch of $5\frac{1}{2}$ yards in length, 40 of them is a rood, and 4 roods an acre: so that a superficies that is 40 perches long and 4 broad, is an acre of land, the acre containing in all 160 square perches.

10. The least denominative part of time is 1 minute; the greatest integer being a year; from whence is produced this following table.

The table of time.

1 Minute	make	1 Minute
60 Minutes	make	1 Hour
24 Hours	make	1 Day natural
7 Days	make	1 Week
4 Weeks	make	1 Month
13 Months, 1 day, 6 hours	make	1 Year.

But

But the year is uſually divided into 12 unequal calendar months; whoſe names, and the number of days they contain, follow, *viz.*

	Days.
January	31
February	28
March	31
April	30
May	31
June	30
July	31
Auguſt	31
September	30
October	31
November	30
December	31
	365

So that the year containeth 365 days and 6 hours; but the 6 hours are not reckoned, but only every 4th year, and then there is a day added to the latter end of February, and then it containeth 29 days, and that year is called *leap year*, and containeth 366 days.

And here note, That as the hour is divided into 60 minutes, ſo each minute is ſubdivided into 60 ſeconds, and each ſecond into 60 thirds, and each third into 60 fourths, *&c.*

The tropical year, by the exacteſt obſervation of the moſt accurate aſtronomers, is found to be 365 days, 5 hours, 49 minutes, 4 ſeconds, and 21 thirds.

C H A P. III.

Of the ſpecies or kinds of arithmetic.

1. ARithmetic is either natural, artificial, analytical, algebraical, lineal, or inſtrumental.

2. Natural arithmetic is that which is performed by the numbers themſelves. And this is either poſitive or negative. Poſitive, which is wrought by certain infallible numbers propounded. And this is either ſingle or comparative. Single, which conſidereth the nature of numbers ſimply by themſelves; and comparative, which is wrought by numbers as they have relation to one another.

ther. And the negative part relates to the rule of falſe.

3. Artificial (by ſome called logarithmetical) arithmetic, is that which is performed by artificial or borrowed numbers invented for that purpoſe, which are called *logarithms*.

4. Analytical arithmetic is that which ſhews how from a thing unknown to find truly that which is ſought, always keeping the ſpecies without change.

5. Algebraical arithmetic is an obſcure and hidden art of accounting by numbers, in reſolving of hard queſtions.

6. Lineal arithmetic is that which is performed by lines fitted to proportion, and geometrical projections.

7. Inſtrumental arithmetic is that which is performed by inſtruments fitted with circular and right lines of proportion, by the motion of an index, or otherwiſe.

8. The parts of ſingle arithmetic are, numeration, and the extraction of roots.

9. Numeration is that which, by certain known numbers propounded, diſcovers another number unknown.

10. Numeration hath four ſpecies, *viz.* addition, ſubtraction, multiplication, and diviſion.

C H A P. IV.

Addition of whole numbers.

1. ADdition is the reduction of two or more numbers of like kind together into one ſum or total; or it is that by which divers numbers are added together, to the end that the ſum or total value of them all may be diſcovered.

The firſt number in every addition is called *the addible number*; the other, *the number* or *numbers added*; and the number invented by the addition, is called the *aggregate* or *ſum*, containing the value of the addition.

The collation of the numbers, is the right placing the numbers given reſpectively to each denomination; and the operation is the artificial adding of the numbers

bers given together, in order to the finding out of the aggregate or ſum.

2. In addition, place the numbers given reſpectively the one above the other, in ſuch ſort, that the like degree, place, or denomination, may ſtand in the ſame ſeries, *viz.* units under units, tens under tens, hundreds under hundreds, *&c.*; pounds under pounds, ſhillings under ſhillings, pence under pence, *&c.*; yards under yards, feet under feet, *&c.*

3. Having thus placed the numbers given, (as before directed), and drawn a line under them, add them together, beginning with the leſſer denomination, *viz.* at the right hand, and ſo on, ſubſcribing the ſum under the line reſpectively. As for example:

Let there be given 3352, and 213, and 133, to be added together. I ſet the units in each particular number under each other, and ſo likewiſe the tens under the tens, *&c.* and draw a line under them, as in the margin. Then I begin at the place of units, and add them together upwards; ſaying, 3 and 3 are 6, and 2 makes 8; which I ſet under the line, and under the figures added together. Then I proceed to the next place, being the place of tens, and add them up in the ſame manner as I did the place of units; ſaying, 3 and 1 are 4, and 5 are 9; which I likewiſe ſet under the line reſpectively. Then I go to the place of hundreds, and add them up as I did the other; ſaying, 1 and 2 are 3, and 3 are 6; which I alſo ſet under the line. And, laſtly, I go to the place of thouſands, and, becauſe there are no other figures to add to the 3, I ſet it under the line in its reſpective place; and ſo the work is finiſhed; and I find the ſum of the three given numbers to be 3698.

$$\begin{array}{r}3352\\213\\133\\\hline 3698\end{array}$$

4. But if the ſum of the figures of any ſeries exceedeth ten, or any number of tens, ſubſcribe under the ſame the exceſs above the tens; and for every ten carry one to be added to the next ſeries towards the left hand, and ſo go on, till you have finiſhed your addition; always remembering, that how great ſoever the ſum of the figures of the laſt ſeries is, it muſt all be ſet down under the line reſpectively. So 3678 being given to be added

to 2357, I ſet them down as is before directed, and as you ſee in the margin, with a line drawn under them. Then I begin and add them together; ſaying, 7 and 8 are 15; which is 5 above 10: wherefore I ſet 5 under the line, and carry 1 for the 10, to be added to the next ſeries; ſaying, 1 that I carried and 5 is 6, and 7 are 13; wherefore I ſet down 3, and carry 1 for the 10 to the next ſeries. Then I ſay, 1 that I carried and 3 are 4, and 6 are 10. Now, becauſe it comes to juſt 10, and no more, I ſet 0 under the line, and carry 1 for the 10 to the next, and ſay, 1 that I carried and 2 are 3, and 3 are 6; which I ſet down in its reſpective place. Thus the addition is ended, and the total ſum of theſe numbers is found to be 6035. Several examples of this kind follow.

3678
2357
6035

Numbers to be added	354867
	573846
	785946
	347205
Sum - -	2061864

Numbers to be added	748647	Numbers to be added	45346
	465834		38074
	76483		8437
	648300		923
			76
Sum - -	1939264	Sum - - -	92856

5. If the numbers given to be added are contained under divers denominations, as of *pounds*, *ſhillings*, *pence*, and *farthings*; or of *tuns*, *hundreds*, *quarters*, *pounds*, &c.; then, in this caſe, having diſpoſed of the numbers, each denomination under other of the like kind; beginning at the leaſt denomination, (minding how many of one denomination do make an integer in the next), and having added them up; for every integer of the next greater denomination that you find therein contained, bear an unit in mind to be added to the ſaid next greater denomination, expreſſing the exceſs reſpectively under the line. Proceed in this manner until your addition be finiſhed. The following example will make the rule plain to the learner.

learner. Thus these following sums being given to be added, *viz.* 136 l. 13 s. 4 d. 2 qrs. and 79 l. 7 s. 10 d. 3 qrs. and 33 l. 18 s. 9 d. 1 qr. also 15 l. 9 s. 5 d. 0 qrs.; the numbers being disposed according to order, will stand as in the margin. Then I begin at the denomination of farthings, and add them up; saying, 1 and 3 are 4, and 2 makes 6. Now I consider that 6 farthings are 1 penny and 2 farthings; wherefore I set down the 2 farthings in its place under the line, and keep 1 in mind to be added to the next denomination of pence. Then I go on, saying, 1 that I carried and 5 are 6, and 9 are 15, and 10 are 25, and 4 are 29. Now, I consider that 29 pence are 2 shillings and 5 pence; therefore I set the 5 pence in order under the line, and keep 2 in mind for the 2 shillings, to be added to the shillings. Then I go on, saying, 2 that I carried and 9 are 11, and 18 are 29, and 7 are 36, and 13 are 49. Then I consider that 49 shillings are 2 pounds and 9 shillings; wherefore I set the 9 shillings under the line, and carry the 2 for the 2 pounds to the next and last denomination of pounds; and proceed, saying, 2 that I carried and 5 make 7, and 3 are 10, and 9 are 19, and 6 are 25. Then I set down 5, and carry 2 for the two tens, and proceed; saying, 2 that I carried and 1 is 3, and 3 are 6, and 7 are 13, and 3 make 16. And I set down 6, and carry 1 for the 10, and go on; saying, 1 that I carried and 1 are 2; which I set in its place under the line, and the work is finished. And thus I find the sum of the foresaid numbers to be 265 l. 9 s. 5 d. 2 qrs. This to the ingenious practitioner is sufficient. But I shall, for the further illuminating of the weaker apprehensions, explain the operation of another example in Troy weight. And here the learner must take notice of the table of Troy weight mentioned or set down in the third section of the second chapter. The numbers given in this example are, 38 l. 7 oz. 13 p. w. 18 gr. and 50 l. 10 oz. 10 p. w. 12 gr. and 42 lb. 8 oz. 5 p. w. 16 gr. And, in order to the addition thereof, I place them as you see, and proceed to operation; saying,

l.	*s.*	*d.*	*qrs.*
136	13	4	2
79	7	10	3
33	18	9	1
15	9	5	0
265	9	5	2

ing, 16 and 12 are 28, and 18 are 46. Now, becauſe 24 grains make 1 pennyweight, 46 grains are 1 pennyweight and 22 grains; wherefore I ſet down 22, and carry 1 for the pennyweight; and, going on, I ſay, 1 that I carry and 5 makes 6, and 10 are 16, and 13 are 29; which is 1 ounce and 9 pennyweight. I ſet down 9 in its place under the line, and carry 1 to the ounces; ſaying, 1 that I carry and 8 are 9, and 10 are 19, and 7 are 26. And becauſe 26 ounces make 2 pounds 2 ounces, I ſet down 2 for the ounces, and carry 2 to the pounds: going on, 2 that I carried and 2 are 4, and 8 makes 12; that is, 2 and go 1: then 1 I carry and 4 are 5, and 5 are 10, and 3 are 13; which I ſet down as in the margin, and the work is finiſhed: and I find the ſum of the ſaid numbers to amount to 132 lb. 2 oz. 9 p. w. 22 gr. This is ſufficient for the underſtanding of the following examples, or any other that ſhall come to thy view. The way of proving theſe, or any ſum in this rule, is ſhewed immediately after the enſuing examples.

lb.	*oz.*	*pw.*	*gr.*
38	7	13	18
50	10	10	12
42	8	5	16
132	2	9	22

Addition of Engliſh money.

l.	*s.*	*d.*	*qrs.*
436	13	7	1
184	9	10	3
768	17	4	2
564	11	11	0
1954	12	9	2

l.	*s.*	*d.*	*qrs.*
48	15	11	1
76	10	7	3
18	0	5	3
24	19	9	2
168	6	10	1

Addition of Troy weight.

lb.	*oz.*	*pw.*	*gr.*
15	7	13	12
18	6	4	20
11	10	16	18
9	4	10	22
19	11	18	4
22	0	0	0
97	5	4	4

lb.	*oz.*	*pw.*	*gr.*
145	9	12	18
726	8	14	10
389	7	6	13
83	10	16	20
130	0	10	12
74	7	15	0
1550	8	16	1

Addition

Addition of apothecaries weight.

lb.	oz.	dr.	sc.	gr.
48	7	1	0	14
74	5	5	2	10
64	10	7	1	16
17	8	1	0	11
34	9	6	1	9
240	5	6	1	0

lb.	oz.	dr.	sc.	gr.
60	3	4	0	10
48	10	6	0	14
34	8	2	1	15
18	11	2	2	11
160	7	1	2	15
35	2	5	1	7
358	7	7	0	12

Addition of Avoirdupois weight.

Tuns.	C.	qrs.	lb.
75	13	1	15
48	7	3	21
60	11	1	17
21	7	0	25
12	16	0	11
218	17	0	5

lb.	oz.	dr.
36	10	12
22	11	13
11	7	4
15	4	10
20	10	9
106	13	0

Addition of liquid measure.

Tuns.	pipes.	hhds.	gall.
45	1	1	48
15	0	1	17
38	0	0	47
12	1	0	56
21	1	1	18
133	0	1	60

Tuns.	hhds.	gall.	pints.
30	3	40	4
12	0	28	6
47	5	60	5
57	3	22	3
17	0	0	0
168	1	26	2

Addition of dry measure.

Chald.	qrs.	bush.	pec.
48	3	7	3
13	1	4	0
54	0	6	2
16	3	6	1
40	1	0	1
173	3	0	3

qrs.	bush.	pec.	gall.
17	3	1	1
50	1	3	0
14	5	3	1
40	2	0	1
30	0	3	0
152	5	3	1

Addition of long measure.

Yards.	qrs.	nails.	Ells.	qrs.	nails.
35	3	3	56	1	3
14	1	2	12	3	2
74	2	3	48	2	1
38	0	1	59	1	0
30	1	0	74	0	2
15	0	0	17	1	0
208	1	1	260	0	0

Addition of land measure.

Acres.	roods.	perch.	Acres.	roods.	perch.
12	3	18	86	1	36
14	0	24	47	3	24
30	2	19	73	2	18
48	3	30	60	0	7
28	1	38	4	2	8
50	3	26	14	1	14
185	3	35	286	3	27

The proof of Addition.

6. Addition is proved after this manner. When you have found out the sum of the number given, then separate the uppermost line from the rest with a stroke or dash of the pen, and then add them all up again as you did before, leaving out the uppermost line; and having so done, add the new invented sum to the uppermost line you separated; and if the sum of those two lines be equal to the sum first found out, then the work was performed true, otherwise not. As for example: Let us prove the first example of addition of money, whose sum we found to be 265 l. 9 s. 5 d. 2 qrs.; and which

which we prove thus. Having ſeparated the uppermoſt number from the reſt by a line, as you ſee in the margin; then I add the ſame together again, leaving out the ſaid uppermoſt line, and the ſum thereof I ſet under the firſt ſum or true ſum, which doth amount to 128 l. 16 s. 1 d. 0 qrs. then again I add this new ſum to the uppermoſt line that before was ſeparated from the reſt, and the ſum of thoſe two is 265 l. 9 s. 5 d. 2 qrs. the ſame with the firſt ſum, and therefore I conclude that the operation was rightly performed.

l.	s.	d.	qrs.
136	13	04	2
79	07	10	3
33	18	09	1
15	09	05	0
265	09	05	2
128	16	01	0
265	09	05	2

7. The main end of addition in queſtions reſolvable thereby, is to know the ſum of ſeveral debts, parcels, integers, &c. Some queſtions may be theſe that follow.

Queſt. 1. There was an old man whoſe age was required. To which he replied, I have ſeven ſons, each having two years between the birth of each other; and in the 44th year of my age my eldeſt ſon was born, which is now the age of the youngeſt. I demand, What was the old man's age?

Now, to reſolve this queſtion, firſt ſet down the father's age at the birth of his firſt child, which was 44; then the difference between the oldeſt and the youngeſt, which is 12 years; and then the age of the youngeſt, which is 44: and then add them all together; and their ſum is 100, the complete age of their father.

44
12
44
100

Queſt. 2. A man lent his friend at ſeveral times theſe ſeveral ſums, *viz.* at one time 63 l. at another time 50 l. at another time 48 l. at another time 156 l. Now I deſire to know how much was lent him in all?

Set the ſums lent one under another, as you ſee in the margin; and then add them together, and you will find their ſum to amount to 317 l. which is the total of all the ſeveral ſums lent, and ſo much is due to the creditor.

63
50
48
156
317

Queſt.

Quest. 3. From London to Ware is 20 miles, thence to Huntington 29 miles, thence to Stamford 21 miles, thence to Tuxford 36 miles, thence to Weatbridge 25 miles, from thence to York 20 miles. Now I desire to know how many miles it is from London to York, according to this reckoning?

Now, to answer this question, set down the several distances given, as you see in the margin; and add them together, and you will find their sum to amount to 151; which is the true distance in miles between London and York.

20
29
21
36
25
20
151

Quest. 4. There are two numbers, the least whereof is 40, and their difference 14. I desire to know what is the greater number, and also what is the sum of them both? First, set down the least, viz. 40, and 14, the difference; and add them together, and their sum is 54 for the greatest number. Then I set 40 (the least) under 54 (the greatest) and add them together, and their sum is 94, equal to the greatest and least numbers.

	40
	14
Greatest	54
Least	40
Sum	94

CHAP. V.

Of subtraction of whole numbers.

1. Subtraction is the taking of a lesser number out of a greater of a like kind, whereby to find out a third number, being or declaring the inequality, excess or difference between the numbers given. Or, Subtraction is that by which one number is taken out of another number given, to the end that the residue or remainder may be known; which remainder is also called the *rest*, *remainder*, or *difference* of the numbers given.

2. The number out of which subtraction is to be made, must be greater, or at least equal with the other number given. The higher or superiour number is called the

the *major* number; and the lower or inferiour is called the *minor* number; and the operation of ſubtraction being finiſhed, the reſt or remainder is called the *difference* of the numbers given.

3. In ſubtraction, place the numbers given reſpectively the one under the other, in ſuch ſort as like degrees, places, or denominations, may ſtand in the ſame ſeries, *viz.* units under units, tens under tens, pounds under pounds, *&c.* feet under feet, and parts under parts, *&c.* This being done, draw a line underneath, as in addition.

4. Having placed the numbers given as is before directed, and drawn a line under them, ſubtract the lower number (which in this caſe muſt always be leſs than the uppermoſt) out of the higher number, and ſubſcribe the difference or remainder reſpectively below the line; and when the work is finiſhed, the number below the line will give you the remainder.

As for example: Let 364521 be given to be ſubtracted from 795836. I ſet the leſſer under the greater, as in the margin, and draw a line under them; then beginning at the right hand, I ſay, 1 out of 6, and there remains 5, which I ſet in order under the line. Then I proceed to the next, ſaying, 2 from 3 reſts 1; which I note alſo under the line. And thus I go on till I have finiſhed the work. And then I find the remainder or difference to be 431315.

795836
364521
431315

5. But if it ſo happen, as commonly it doth, that the lowermoſt number or figure is greater than the uppermoſt; then, in this caſe, add 10 to the uppermoſt number, and ſubtract the ſaid lowermoſt number from their ſum, and the remainder place under the line; and when you go to the next figure below, pay an unit, by adding it thereto for the 10 you borrowed before, and ſubtract that from the higher number of figures. And thus go on till your ſubtraction be finiſhed. As for example: Let 437503 be given, from whence it is required to ſubtract 153827. I diſpoſe of the numbers as is before directed, and as you ſee in the margin; then I begin, ſaying, 7 from 3 I cannot, but, adding 10 thereto, I ſay, 7 from 13, and there remains 6; which I ſet under the line in order. Then

I

I proceed to the next figure, ſaying, 1 that I borrowed and 2 is 3 from 0 I cannot, but 3 from 10, and there remains 7; which I likewiſe ſet down as before. Then 1 that I borrowed and 8 is 9 from 5 I cannot, but 9 from 15, and there remains 6. Then 1 I borrowed and 3 is 4 from 7, and there remains 3. Then 5 from 3 I cannot, but 5 from 13, and there remains 8. Then 1 I borrowed and 1 are 2 from 4, and there reſts 2. And thus the work is finiſhed. And after theſe numbers are ſubtracted one from another, the inequality, remainder, exceſs, or difference, is found to be 283676. Examples for thy farther experience may be theſe that follow.

437503
153827
283676

From	3469916	From	361576
Take	738642	Take	5864
Reſts	2731274	Reſts	355712

6. If the ſum or number to be ſubtracted is of ſeveral denominations, place the leſſer ſum below the greater, and in the ſame rank and order as is ſhewed in addition of the ſame numbers. Then begin at the right hand, and take the lower number out of the uppermoſt, if it be leſſer; but if it be bigger than the uppermoſt, then borrow an unit from the next greater denomination, and turn it into the parts of the leſs denomination, and add thoſe parts to the uppermoſt, noting the remainder below the line. Then proceed, and pay 1 to the next denomination for that which you borrowed before; and proceed in this order, until the work be finiſhed. An example of this rule may be this that followeth. Let 375 l. 13 s. 7 d. 1 qr. be given, from whence let it be required to ſubtract 57 l. 16 s. 3 d. 2 qrs. In order whereunto I place the numbers as you ſee in the margin. And thus I begin at the leaſt denomination, ſaying, 2 from 1 I cannot, therefore I borrow 1 penny from the next denomination, and turn it into farthings, which is 4; and adding 4 to 1, which is 5, I ſay,

l.	*s.*	*d.*	*q.*
375	13	7	1
57	16	3	2
317	17	3	3

2 from 5, and there remains 3; which I put under the line. Then going on, I ſay, 1 that I borrowed and 3 is 4 from 7, and there reſts 3. Then going on, I ſay, 16 from 13 I cannot, but borrowing 1 pound, and turning it into 20 ſhillings, I add it to 13, and that is 33; wherefore I ſay, 16 from 33, and there remains 17; which I ſet under the line; and go on, ſaying, 1 that I borrowed and 7 is 8 from 5 I cannot, but 8 from 15, and there remains 7; the 1 that I borrowed and 5 is 6 from 7, there reſts 1, and 0 from 3 reſts 3. And the work is done. And I find the remainder or difference to be 317 l. 17 s. 3 d. 3 qrs.

An example of Troy weight may be this. I would ſubtract 17 lb. 10 oz. 11 pw. 20 gr. from 24 lb. 5 oz. 0 pw. 8 gr. I place the numbers according to the rule; and begin, ſaying 20 from 8 I cannot, but borrow 1 pennyweight, which is 24 grains, and add them to 8, and they are 32; wherefore I ſay, 20 from 32 reſts 12.

lb.	*oz.*	*pw.*	*gr.*
24	5	0	8
17	10	11	20
6	6	8	12

Then 1 that I borrowed and 11 is 12 from 0 I cannot, but 12 from 20, borrowing an ounce, which is 20 penny-weight, and there remains 8. Then 1 that I borrowed and 10 is 11 from 5 I cannot, but 11 from 17, and there reſts 6. Then 1 that I borrowed and 7 is 8 from 4 I cannot, but 8 from 14, and there reſts 6. Then 1 that I borrowed and 1 is 2 from 2, and there reſts nothing. So that I find the remainder or difference to be 6 lb. 6 oz. 8 pw. 12 gr.

7. It many times happeneth, that you have many ſums or numbers to be ſubtracted from one number, as ſuppoſe a man ſhould lend his friend a certain ſum of money, and his friend hath paid him part of his debt at ſeveral times. Then, before you can conveniently know what is ſtill owing, you are to add the ſeveral numbers or ſums of payments together, and ſubtract their ſum from the whole debt, and the remainder is the ſum due to the creditor. As ſuppoſe A lendeth to B 564 l. 16 s. 10 d. and B hath repaid him 79 l. 16 s. 8 d. at one time, and 163 l. 18 s. 11 d. at another

other time, and 241 l. 15 s. 8 d. at another time; and you would know how the account ſtandeth between them, or what more is due to A. In order whereunto I firſt ſet down the ſum which A lent, and draw a line underneath it; then under that line I ſet the ſeveral ſums of payment as you ſee in the margin; and having brought the ſeveral ſums of payment into one total, by the fifth rule of the fourth chapter foregoing, I find their ſum amounteth to 485 l. 11 s. 3 d. which I ſubtract from the ſum firſt lent by A, by the ſixth rule of this chapter; and I find the remainder to be 79 l. 5 s. 7 d. and ſo much is ſtill due to A.

	l.	*s.*	*d.*
Lent -	564	16	10
Paid at ſeveral payments.	79	16	8
	163	18	11
	241	15	8
Paid in all	485	11	3
Remains -	79	5	7

When the learner hath good knowledge of what hath been already delivered in this and the foregoing chapters, he will with eaſe underſtand the manner of working the following examples.

Subtraction of money.

	l.	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
Borrowed - -	374	10	3	700	10	11	2
Paid - - -	79	15	11	9	3	11	3
Remains - -	294	14	4	691	6	11	3

	l.	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
Borrowed - -	1000	0	0	711	3	0	0
Paid - - - -	19	0	6	11	13	0	1
Rem. due - -	980	19	6	699	9	11	3

	l.	*s.*	*d.*	*qrs.*
Borrowed - - -	3300	0	0	0
Paid at ſeveral payments	170	10	0	0
	361	13	10	1
	590	3	4	3
	73	4	11	3
Paid in all - -	1195	12	2	3
Remains due - -	2104	7	9	1

Subtraction of Troy weight.

	lb.	*oz.*	*pw.*	*gr.*
Bought - - -	174	0	13	0
Sold - - - -	78	4	16	15
Remains - - -	95	7	16	9

	lb.	*oz.*	*pw.*	*gr.*
Bought - - -	470	10	13	0
Sold at ſeveral times	60	0	0	0
	35	10	18	0
	16	7	9	8
	48	4	0	0
	61	11	19	23
	23	0	0	0
Sold in all - -	245	10	7	7
Remains unſold -	225	0	5	17

Subtraction of apothecaries weight.

	lb.	*oz.*	*dr.*	*ſc.*	*gr.*	*lb.*	*oz.*	*dr.*	*ſc.*	*gr.*
Brought	12	4	3	0	0	20	0	1	0	7
Sold -	8	5	1	1	15	10	0	1	2	12
Remains	3	11	1	1	5	9	11	7	0	15

Subtraction of Avoirdupois weight.

	C.	qrs.	lb.	Tu.	C.	qrs.	lb.	oz.	dr.
Bought	35	0	15	5	7	1	10	10	5
Sold	16	2	20	3	17	1	16	9	13
Remains	18	1	23	1	9	3	22	0	8

Subtraction of liquid measure.

	Tuns.	hhd.	gall.	Tuns.	hhd.	gall.	pints.
Bought	40	1	30	60	3	42	4
Sold	16	1	40	15	3	46	6
Remains	23	3	53	44	3	58	6

Subtraction of dry measure.

	Chal.	qrs.	bush.	pec.	Chald.	qrs.	bush.	pec.
Bought	100	0	0	0	73	2	3	2
Sold	54	1	4	3	46	2	3	3
Remains	45	2	3	1	26	3	7	3

Subtraction of long measure.

	Yards.	qrs.	nail.	Yards.	qrs.	nail.
Bought	160	0	0	344	0	1
Sold	64	1	2	177	1	3
Remains	95	2	2	166	2	2

Subtraction of land measure.

	Acres.	rood.	perch.	Acres.	roods.	perch.
Bought	140	2	13	600	0	0
Sold	70	3	12	54	0	16
Remains	69	3	1	545	3	34

The

The proof of ſubtraction.

8. When your ſubtraction is ended, if you deſire to prove the work, whether it be true or no, then add the remainder to the minor number; and if the aggregate of theſe two be equal to the major number, then is your operation true; otherwiſe falſe. Thus let us prove the firſt example of the fifth rule of this chapter, where, after ſubtraction is ended, the numbers ſtand as in the margin; the remainder or difference being 283676. Now, to prove the work, I add the ſaid remainder 283676 to the minor number 153827, by the fourth rule of the foregoing chapter, and I find the ſum, or aggregate, to be 437503, equal to the major number, or number from whence the leſſer is ſubtracted. Behold the work in the margin.

437503
153827
283676
437503

The proof of another example, may be of the firſt example of the ſixth rule of this chapter, where it is required to ſubtract 57 l. 16 s. 3 d. 2 qrs. from 375 l. 13 s. 7 d. 1 qr. and by the rule I find the remainder to be 317 l. 17 s. 3 d. 3 qrs. Now, to prove it, I add the ſaid remainder 317 l. 17 s. 3 d. 3 qrs. to the minor number 57 l. 16 s. 3 d. 2 qrs. and their ſum is 375 l. 13 s. 7 d. 1 qr equal to the major number, which proves the work to be true; but if it had happened to have been either more or leſs than the ſaid major number, then the operation had been falſe.

l.	*s.*	*d.*	*q.*
375	13	7	1
57	16	3	2
317	17	3	3
375	13	7	1

9. The general effect of ſubtraction, is, to find the difference or exceſs between two numbers; and the reſt, when a payment is made in part of a greater ſum; the date of books printed, the age of any thing, by knowing the preſent year, and the year wherein they were made, created, or built; and ſuch like.

The queſtions appropriated to this rule, are ſuch as follow.

Queſt. 1. What difference is there between one thing of 125 foot long, and another of 66 foot long?

To reſolve this queſtion, I firſt ſet down the major or greater number 125, and under it the minor or leſſer number 66, as is directed in the third rule of this chapter; and according to the fourth rule of the ſame, I ſubtract the minor from the major; and the remainder, exceſs, or difference, I find to be 59. See the work in the margin.

125
66
59

Queſt. 2. A gentleman oweth a merchant 365 l. whereof he hath paid 278 l. what more doth he owe?

To give an anſwer to this queſtion, I firſt ſet down the major number 365 l. and under it I place 278 the minor, and ſubtract the one from the other; whereby I diſcover the exceſs, difference, or remainder, to be 87; and ſo much is ſtill due to the creditor, as *per* margin.

365
278
87

Queſt. 3. An obligation was written, a book printed, a child born, a church built, or any other thing made in the year of our Lord 1572; and now we account the year of our Lord 1687. The queſtion is to know the age of the ſaid things; that is, how many years are paſſed ſince the ſaid things were made? I ſay, if you ſubtract the leſſer number 1572, from the greater 1687, the remainder will be 115; and ſo many years are paſſed ſince the making of the ſaid things, as by the work in the margin.

1687
1572
115

Queſt. 4. There are three towns that lie in a ſtraight line, *viz.* London, Huntington, and York. Now, the diſtance between the fartheſt of theſe towns, *viz.* London and York, is 151 miles, and from London to Huntington is 49 miles, I demand how far it is from Huntington to York.

To reſolve this queſtion, ſubtract 49, the diſtance between London and Huntington, from 151, the diſtance between London and York, and the remainder is 102 for the true diſtance between Huntington and York. See the work in the margin.

151
49
102

CHAP.

CHAP. VI.

Of Multiplication of whole numbers.

1. MUltiplication is performed by two numbers of like kind for the production of a third, which ſhall have ſuch reaſon to the one, as the other hath to unit; and, in effect, is a moſt brief and artificial compound addition of many equal numbers of like kind into one ſum. Or, multiplication is that by which we multiply two or more numbers, the one into the other, to the end that their product may come forth, or be diſcovered.

Or, multiplication is the increaſing of any one number by another, ſo often as there are units in that number by which the other is increaſed; or, by having two numbers given, to find a third which ſhall contain one of the numbers as many times as there are units in the other.

2. Multiplication hath three parts. *1ſt*, The multiplicand, or number to be multiplied. *2dly*, The multiplier, or number given, by which the multiplicand is to be multiplied. And, *3dly*, The product or number produced by the other two, the one being multiplied by the other. As if 8 were given to be multiplied by 4, I ſay, 4 times 8 is 32; here 8 is the multiplicand, and 4 is the multiplier, and 32 is the product.

$$\begin{array}{r} 8 \\ 4 \\ \hline 32 \end{array}$$

3. Multiplication is either ſingle, by one figure; or compound, that conſiſts of many.

Single multiplication is ſaid to conſiſt of one figure, becauſe the multiplicand and multiplier conſiſt each of them of a digit and no more; ſo that the greateſt product that can ariſe by ſingle multiplication, is 81, being the ſquare of 9. And compound multiplication is ſaid to conſiſt of many figures, becauſe the multiplicand or multiplier conſiſts of more places than one: as if I were to multiply 436 by 6, it is called compound, becauſe the multiplicand 436 is of more places than one, *viz.* three places.

4 The learner ought to have all the varieties of single multiplication by heart, before he can well proceed any farther in this art; it being of most excellent use, and none of the following rules in arithmetic but what have a principal dependence thereupon; which may be learned by the following table.

MULTIPLICATION-TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the preceding table is this. In the uppermost line or column you have expressed all the digits, from 1 to 9; and likewise beginning at 1, and going downwards in the side column, you have the same; so that if you would know the product of any two single numbers multiplied by one another, look for one of them, which you please, in the uppermost column, and for the other in the side-column; and runing your eye from each figure along the respective columns, in the common angle, or place where these two columns meet, there is the product required. As for example: I would know how much is 8 times 7. First, I look for 8 in the uppermost column, and 7 in the side column; then do I cast my eye from 8, along the

the column downwards from the ſame, and likewiſe from 7 in the ſide column, I caſt my eye from thence toward the right hand, and find it to meet with the firſt column at 56; ſo that I conclude 56 to be the product required. It would have been the ſame if you had looked for 7 in the top, and 8 on the ſide. The like is to be underſtood of any other ſuch numbers. The learner being perfect herein, it will be neceſſary to proceed.

5. In compound multiplication, if the multiplicand conſiſts of many places, and the multiplier of but one figure; firſt ſet down the multiplicand, and under it place the multiplier in the place of units, and draw a line underneath them. Then begin, and multiply the multiplier into every particular figure of the multiplicand, beginning at the place of units; and ſo proceed towards the left hand, ſetting each particular product under the line, in order as you proceed: but if any of the products exceed 10, or any number of tens, ſet down the exceſs, and for every 10 carry an unit to be added to the next product; always remembering to ſet down the total product of the laſt figure. Which work being finiſhed, the ſum or number placed under the line ſhall be the true and total product required. As for example: I would multiply 478 by 6. Firſt I ſet down 478, and underneath it 6, in the place of units, and draw a line underneath them, as in the margin. Then I begin, ſaying, 6 times 8 is 48, which is 8 above 4 tens; therefore I ſet down 8, the exceſs, and bear 4 in mind for the 4 tens. Then I proceed, ſaying, 6 times 7 is 42, and four that I carried is 46; I then ſet down 6, and carry 4: and go on, ſaying, 6 times 4 is 24, and 4 that I carried is 28; and becauſe it is the laſt figure, I ſet it all down. And ſo the work is finiſhed; and the product is found to be 2868, as was required.

$$\begin{array}{r} 478 \\ 6 \\ \hline 2868 \end{array}$$

6. When in compound multiplication, the multiplier conſiſteth of divers places, then begin with the figure in the place of units in the multiplier, and multiply it into all the figures in the multiplicand, placing the product below the line, as was directed in the laſt example. Then begin with the figure of the ſecond place of the multiplier,

tiplier, (*viz.*) the place of tens, and multiply it likewise into the whole multiplicand, as you did the first figure, placing its product under the product of the first figure. Do in the same manner by the third, fourth, and fifth, &c. until you have multiplied all the figures of the multiplier particularly into the whole multiplicand, still placing the product of each particular figure under the product of its preceding figure; herein observing the following caution.

A caution. In the placing of the product of each particular figure of the multiplier, you are not to follow the 2d rule of the fourth chapter, *viz.* to place units under units, and tens under tens, &c. but to put the figure or cipher in the place of units of the second line under the second figure or place of tens in the line above it, and the figure or cipher in the place of units of the third line, under the place of tens in the second line, &c.; observing this order till you have finished the work; still placing the first figure of every line or product under the second figure, or place of tens in that which is above it. And having so done, draw a line under all these particular products, and add them together; so shall the sum of all these products be the total product required.

As if it were required to multiply 764 by 27, I set them down the one under the other, with a line drawn underneath them. Then I begin, saying, 7 times 4 is 28; then I set down 8, and carry 2. Then I say, 7 times 6 is 42, and 2 that I carried is 44, that is 4 and go four. Then 7 times 7 is 49, and 4 that I carry is 53, which I set down; because I have not another figure to multiply. Thus I have done with the 7. Then I begin with the 2, saying, 2 times 4 is 8, which I set down under (4) the second figure or place of tens in the line above it, as you may see in the margin. Then I proceed, saying, 2 times 6 is 12, that is 2, and carry 1. Then 2 times 7 is 14, and 1 that I carry is 15, which I set down; because it is the product of the last figure: so that the product of 764 by 7, is 5348; and by 2, is 1528, which being placed the one under the other, as is before directed, as you see in the margin,

```
  764
   27
 ----
 5348
1528
 ----
20628
```

and

and a line drawn under them, and they added together respectively, make 20628, the true product required, being equal to 27 times 764.

Another example may be this. Let it be required to multiply 5486 by 465: I dispose of the multiplicand and multiplier according to the rule, and begin, multiplying the first figure of the multiplier, which is 5, into the whole multiplicand, and find the product is 27430. Then I proceed, and multiply the second figure (6) of the multiplier into the multiplicand, and find the product to amount to 32916; which is subscribed under the other product respectively. Then do I multiply the third and last figure (4) of the multiplier into the multiplicand, and the product is 21944; which is likewise placed under the second line respectively. Then I draw a line under the said products, being placed the one under the other according to the rule, and add them together, and the sum is 2550990, the true product sought; being equal to 5486 times 465, or 465 times 5486.

5486
465
27430
32916
21944
2550990

More examples in this rule are these following.

430865	6400758
4739	37496
3877785	38404548
1292595	57606822
3016055	25603032
1723460	44805306
	19202274
2041869235	240002821968

Compendium in multiplication.

7. Although the former rules are sufficient for all cases in multiplication, yet because in the work of multiplication many times great labour may be saved, I shall acquaint the learner with some compendiums in order thereto, *viz.* If the multiplicand or multiplier, or both of them, end with ciphers; then in your multiplying you may neglect

th

the ciphers, and multiply only the ſignificant figures: and to the product of thoſe ſignificant figures, add ſo many ciphers as the numbers given to be multiplied did end with *; that is, annex them on the right hand of the ſaid product, ſo ſhall that give you the true product required. As if I were to multiply 32000 by 4300, I ſet them down in order to be multiplied, as you ſee in the margin; but neglecting the ciphers in both numbers, I only multiply 32 by 43, and the product I find to be 1376; to which I annex the 5 ciphers that are in the multiplicand and multiplier, and then it makes 137600000 for the true product of 32000 by 4300.

```
     32000
     4300
    ------
     96
    128
    ---------
    137600000
```

8. If in the multiplier, ciphers are placed between ſignificant figures, then multiply only by the ſignificant figures, neglecting the ciphers †. But here ſpecial notice is to be taken of the true placing of the firſt figure after the neglect of ſuch cipher or ciphers: and therefore you muſt obſerve in what place of the multiplier the figure you multiply by ſtandeth, and ſet the firſt figure of that product under the ſame place of the product of the firſt figure of your multiplier. As for example: Let it be required to multiply 371568 by 40007. Firſt I multiply the multiplicand by 7, and the product is 2600976. Then, neglecting the ciphers, I multiply by 4, and that product is 1486272. Now I conſider that 4 is the 5th figure in the multiplier; therefore I place 2 (the firſt figure of the product by 4) under the 5th place of the 1ſt product by 7, and the reſt in order: and having added them together, the total product is found to be 14865320976. Other examples in this rule are theſe following.

```
         371568
          40007
    -----------
        2600976
    1486272
    -----------
    14865320976
```

* *Si ex numeris propoſitis unus vel uterque adjunctos habeat ad dexteram circulos, omiſſis circulis, fiat ipſorum numerorum multiplicatio, et facto denum t s inſuper integrorum, loci, accenſeantur quot ſunt omiſſi circuli in utroque factore.* Clavis Mat. cap. iv. 3.

† *Si in medio multiplicantis loco circulus fuerit, ille negligitur.* Alſted. cap. vi. de arith.

327586

```
   327586          7864371
     6030            20604
  -------         --------
   982758         31457484
 1965516         47186226
 ----------     15728742
 1975343580     ------------
                162037500084
```

9. If you are to multiply any number by an unit with ciphers *(viz.)* by 10, 100, 1000, *&c.* then annex ſo many ciphers before the multiplicand, and the number, when the ciphers are annexed, is the product required. As if you would multiply 428 by 100, annex two ciphers to 428, and it is 42800. If it were required to multiply 100 by 10000, annex four ciphers, and it gives 1000000 for the product required.

The proof of multiplication.

10. Multiplication is proved by diviſion; and, to ſpeak truth, all other ways are falſe *; and therefore it will be moſt convenient, in the firſt place, to learn diviſion, and by that to prove multiplication. There is a way (at this day generally uſed in ſchools) to prove multiplication, which is this. Firſt add all the figures in the multiplicand together, as if they were ſimple numbers, caſting away the nines as oft as it comes to ſo much, noting the remainder at laſt, which in this caſe cannot be ſo much as 9: caſt likewiſe the nines out of the multiplier as you did out of the multiplicand, and note the remainder. Then multiply the remainders, one by the other, and caſt the mines out of the product, obſerving the remainder. And, laſtly, caſt the nines out of the total product, and if this remainder be equal to the remainder laſt found, then they conclude the work to be rightly performed. But there may be given a thouſand nay infinite falſe products in multiplication, which after this manner may be proved to be true: and therefore this way of proving doth not deſerve any example. But we ſhall defer the proof of

* *Namque non eſt quod aliam expectes examinandi viam; nam aliæ vulgares et falſæ ſunt, et nullo innixæ fundamento.* Gemma Friſius.

this

this rule till we come to prove division, and then we shall prove them both together.

11. The general effect of multiplication is contained in the definition of the same; which is, to find out a third number, so often containing one of the two given numbers, as the other containeth unity.

The second effect is, by having the length and breadth of any thing, (as a parallelogram or long plane), to find the superficial contents of the same; and by having the superficial content of the base, and the length, to find out the solidity of any parallelopipedon, cylinder, or other solid figures.

The third effect is, by the contents, price, value, buying, selling, expense, wages, exchange, simple interest, gain or loss of any one thing, be it money, merchandise, *&c.* to find out the value, price, expense, buying, selling, exchange, or interest of any number of things of like name, nature, and kind.

The fourth effect is not much unlike the other; by the contents, value, or price of any one part of any thing denominated, to find out the contents, value, or price of the whole thing, all the parts into which the whole is divided, multiplying the price of one of those parts.

The fifth effect is, to aid, to compound, and to make other rules; as chiefly the rule of proportion, called the *golden rule* or *rule of three*. Also by it, things of one denomination are reduced to another.

If you multiply any number of integers, or the price of the integer, the product will discover the price of the quantity, or number of integers given.

In a rectangular solid, if you multiply the breadth of the base by the depth, and that product by the length, this last product will discover the solidity or content of the same solid.

Some questions proper to this rule, may be these following.

Quest. 1. What is the content of a square piece of ground, whose length is 28 perches, and breadth 13 perches?

Answer 364 square perches: for multiplying 28 the length by 13 the breadth, the product is so much.

Quest.

Quest. 2. There is a ſquare battle, whoſe flank is 47 men, and the files 19 deep; what number of men doth that battle contain? *Facit* 893: for multiplying 47 by 19, the product is 893.

Queſt. 3. If any one thing coſt 4 ſhillings, what ſhall 9 things coſt? *Anſ.* 36 ſhillings: for multiplying 4 by 9, the product is 36.

Queſt. 4. If a piece of money or merchandiſe be worth or coſt 17 ſhillings, what ſhall 19 ſuch pieces of money or merchandiſe coſt? *Facit* 323 ſhillings, which is equal to 16 l. 3 s.

Queſt. 5. If a ſoldier or ſervant get or ſpend 14 s. *per* month, what is the wages or charges of 49 ſoldiers or ſervants for the ſame time? Multiply 49 by 14, the product is 686 s. or 34 l. 6 s. for the anſwer.

Queſt. 6. If in a day there are 24 hours, how many hours are there in a year, accounting 365 days to conſtitute the year? *Facit* 8760 hours: to which if you add the 6 hours over and above 365 days, as there is in a year, then it will be 8766 hours. Now, if you multiply this 8766 by 60, the number of minutes in an hour, it will produce 525960, the number of minutes in a year.

C H A P. VII.

Of diviſion of whole numbers.

1. DIviſion is the ſeparating or parting of any number or quantity given into any parts aſſigned; or to find how often one number is contained in another; or from any two numbers given, to find a third that ſhall conſiſt of ſo many units, as the one of thoſe two given numbers is comprehended or contained in the other.

2. Diviſion hath three parts or numbers remarkable, *viz.* firſt, the dividend; ſecondly, the diviſor; thirdly, the quotient. The dividend is the number given to be parted or divided. The diviſor is the number given by which the dividend is divided; or it is the number which ſheweth how many parts the dividend is to be divided into. And the quotient is the number produced by

the division of the two given numbers, the one by the other.

So 12 being given to be divided by 3, or into three equal parts, the quotient will be 4; for 3 is contained in 12 four times: where 12 is the dividend, and 3 is the divisor, and 4 is the quotient.

3. In division set down your dividend, and draw a crooked line at each end of it; and before the line at the left hand place the divisor, and behind that on the right hand place the figures of the quotient, as in the margin; where it is required to divide 12 by 3. First I set down 12 the dividend, and on each side of it do I draw a crooked line, and before that on the left hand do I place 3 the divisor. Then do I seek how often 3 is contained in 12; and because I find it 4 times, I put 4 behind the crooked line on the right hand of the dividend, denoting the quotient.

3) 12 (4

4. But if, when the divisor is a single figure, the dividend consisteth of two or more places; then, having placed them for the work, as is before directed, put a point under the first figure on the left hand of the dividend, provided it be bigger than, or equal to the divisor; but if it be less than the divisor, then put a point under the second figure from the left hand of the dividend: which figures, as far as the point goeth from the left hand, are to be reckoned by themselves as if they had no dependence upon the other part of the dividend; and, for distinction's sake, may be called the *dividual*. Then ask how often the divisor is contained in the dividual; placing the answer in the quotient. Then multiply the divisor by the figure that you placed in the quotient, and set the product thereof under your dividual. Then draw a line under the product, and subtract the said product from the dividual, placing the remainder under the said line. Then put a point under the next figure in the dividend on the right hand of that to which you put the point before, and draw it down, placing it on the right hand of the remainder which you found by subtraction; which remainder, with the said figure annexed to it, shall be a new dividual. Then seek again how often the divisor is contained in this new dividual; and put the answer in the

the quotient on the right hand of the figure which you put there before. Then multiply the divisor by the last figure that you put in the quotient, and subscribe the product under the dividual, and make subtraction; and to the remainder draw down the next figure from the grand dividend, (having first put a point under it), and put it on the right hand of the remainder for a new dividual, as before, *&c.* and proceed thus till the work is finished.

Observing this general rule in all kinds of division.

First, To seek how often the divisor is contained in the dividual. Then, having put the answer in the quotient, multiply the divisor thereby, and subtract the product from the dividual. An example or two will make the rule plain. Let it be required to divide 2184 by 6. I dispose the numbers given as is before directed, and as you see in the margin, in order to the work.

6) 2184 (3

Then, because 6 the divisor is more than 2 the first figure of the dividend, I put a point under 1 the second figure, which makes 21 for the dividual. Then do I ask how often 6 the divisor is contained in 21; and because I cannot have it more than 3 times, I put 3 in the quotient; and thereby do I multiply the divisor (6), and the product is 18; which I set in order under the dividual, and subtract it therefrom, and the remainder (3) I place in order under the line, as you see in the margin.

```
6) 2184 (3
    .
   18
   --
    3
```

Then do I make a point under the next figure of the dividend, being 8, and draw it down, annexing it to the remainder 3; so have I 38 for a new dividual. Then do I seek how often 6 is contained in 38; and because I cannot have it more than 6 times, I put 6 in the quotient; and thereby do I multiply the divisor (6), and the product (36) I put under the dividual (38), and subtract it therefrom, and the remainder (2) I put under the line, as you see in the margin.

```
6) 2184 (36
    ..
   18
   --
    38
    36
    --
     2
```

Then do I put a point under the next (and laſt) figure of the dividend, (bing 4), and draw it down to the remainder (2); and putting it on the right hand thereof, it maketh 24 for a new dividual. Then I ſeek how often 6 is contained in 24; and the anſwer is 4; which I put in the quotient, and multiply the diviſor (6) thereby; and the product (24) I put under the dividual (24), and ſubtract it therefrom, and the remainder is 0. And thus the work is finiſhed; and I find the quotient to be 364; that is, 6 is contained in 2184 juſt 364 times, or 2184 being divided into 6 equal parts, 364 is one of thoſe parts.

```
6) 2184 (364
    . .
   18
   ---
    38
    36
    ---
     24
     24
     ---
     (0)
```

Again, if it were required to divide 2646 by 7, or into 7 equal parts, the quotient will be found to be 378; as appeareth by the operation on the margin.

```
7) 2646 (378
   . . .
   21
   ---
    54
    49
    ---
     56
     56
     ---
     (0)
```

So if it were required to divide 946 by 8, the quotient will be found to be 118, and 2 remaining after diviſion is ended. The work appeareth on the margin.

```
8) 946 (118
   . . .
   8
   ---
   14
    8
   ---
    66
    64
    ---
    (2)
```

Many times the dividend cannot exactly be divided by the diviſor, but ſomething will remain; as in the laſt example;

example; where 946 was given to be divided by 8, the quotient was 118, and there remaineth 2 after the division is ended. Now, what is to be done in this case with the remainder, the learner shall be taught when we come to treat of the reducing (or reduction) of fractions.

And here note, that if, after your division is ended, any thing do remain, it must be less than your divisor; for otherwise your work is not rightly performed.

Other examples are such as follow.

```
8) 73464 (9183          9) 13758 (1528
    ....                    ....
   72                       9
   --                       --
    14                      47
     8                      45
    --                      --
     66                      25
     64                      18
     --                      --
      24                      78
      24                      72
      --                      --
      (0)                     (6)
```

5. But if the divisor consisteth of more places than one, then chuse so many figures from the left side of the dividend for a dividual as there are figures in the divisor; and put a point under the farthest figure of that dividual to the right hand, and seek how often the first figure on the left side of the divisor is contained in the first figure on the left side of the dividual, and place the answer in the quotient, and thereby multiply your divisor, placing your product under your dividual, and subtract it therefrom, placing the remainder below the line. Then put a point under the next figure in the dividend, and draw it down to the said remainder, and annex it on the right side thereof, which makes a new dividual; and proceed as before till the work is finished.

And if it so happen, that, after you have chosen your first dividual, as is before directed, you find it to be less than the divisor; then put a point under the figure more near to the right hand, and seek how often the first figure on the left side of the divisor is contained in the two first figures on the left side of the dividual, and place the answer in the quotient; by which multiply the divisor, and place the product thereof in order under the dividual, and subtract it therefrom, and then proceed as before.

Always remembering, that in all cases of division, if, after you have multiplied your divisor by the figure first placed in the quotient, the product be greater than the dividual, then you must cancel that figure in the quotient, and, instead thereof, put a figure less by an unit, (or one), and multiply the divisor thereby; and, if still the product be greater than the dividual, make the figure in the quotient yet less by an unit. And thus do, until your product be less than the dividual, or at the most equal thereto, and then make subtraction, *&c.*

So, if you would divide 9464 by 24, the quotient will be found to be 394. I first put down the given number, as is before directed in the third rule. Now, because my divisor consisteth of two figures, I therefore put a point under the second figure from the left hand of my dividend, which here is 4; wherefore I seek how often 2 (the first figure on the left side of the divisor) is contained in 9, (the like first in the dividual); the answer is 4; which I put in the quotient, and thereby multiply all the divisor, and find the product to be 96, which is greater than the dividual 94; wherefore I cancel the 4 in the quotient, and, instead thereof, I put 3, (an unit less), and by it multiply the divisor 24, and the product is 72, which I subtract from 94 the dividual, and the remainder is 22. Then do I make a point under the next figure 6 in the dividend, and draw it down, and place it on the right side of the remainder 22, and it makes 226 for a new dividual. Now, because the dividual 226 consisteth of a

```
24) 9464 ( 3
     . .   4
    72
    --
    22
```

figure

figure more than the divisor, therefore I seek how often 2 (the first figure of the divisor) is contained in 22, the two first of the dividual: I say 9 times: wherefore I put 9 in the quotient, and thereby multiply the divisor 24; the product (216) I place under the dividual 226, and subtract it from it, and there remaineth 10.

```
24) 9464 (39
     ..
    72
    ---
    226
    216
    ---
     10
```

Then I go on, and make a point under the next and last figure (4) in the dividend, and draw it down to the remainder 10, and it makes 104 for a new dividual: which is also a figure more than the divisor; and therefore I seek how often 2 is contained in 10: I answer 5 times. But multiplying my divisor by 5, the product is 120; which is greater than the dividual: and therefore I make it but 4; and by it multiply the divisor; and the product is 96; which being placed under, and subtracted from the dividual, there remaineth 8. And thus the whole work of this division is ended; and I find, that 9464 being divided by 24, or into 24 equal parts, is found to be 394, as was said before, and the remainder is 8; as you see in the work on the margin.

```
24) 9464 (394
     ...
    72
    ---
    226
    216
    ---
     104
      96
     ---
     (8)
```

Another example may be this. Let there be required the quotient of 1183653 divided by 385. First, I dispose of the numbers, in order to their dividing; and because 118, the three first figures of the dividend, is less than the divisor 385, I therefore make a point under the fourth figure, which is 3, and see how often 3 (the first figure of the divisor) is contained in 11: the answer is 3; which I put in the quotient, and thereby multiply the divisor 385, and the product is 1155; which I subtract from the dividual 1183, and there remains 28. Then (as before) I draw down the next figure, which is 6, and place it before the remainder

```
385) 1183653 (3
        .
     1155
     ----
       28
```

mainder 28; so have I 268 for a new dividual; and because it hath no more figures than the divisor, I seek how often 3 (the first figure in the divisor) is contained in 2, (the first figure of the dividual), and the answer is 0; for a greater number cannot be contained in a lesser; wherefore I put 0 in the quotient: and thereby (according to the 5th rule) I should multiply my divisor; but if I do, the product will be 0; and 0 subtracted from the dividual 286, the remainder is the same. Wherefore I draw down the next figure (5) from the dividend, and put it before the said remainder 286: so have I 2865 for a new dividual; and because it consisteth of four places, *viz.* a place more than the divisor, I seek how often 3 (the first figure of the divisor) is contained in 28 (the two first of the dividual), and I say, there is 9 times 3 in 28; but multiplying my whole divisor (385) thereby, I find the product to be 3465, which is greater than the dividual 2865: wherefore I chuse 8, which is less by an unit than 9; and thereby I multiply my divisor 385, and the product is 3080; which still is greater than the said dividual; wherefore I chuse another number yet an unit less, *viz.* 7, and having multiplied my divisor thereby, the product is 2695; which is less than the dividual 2865: wherefore I put 7 in the quotient, and subtract 2695 from the dividual 2865, and there remains 170. Then I draw down the last figure (3) in the dividend, and place it before the said remainder 170, and it makes 1703 for a new dividual. Then (for the reason above said) I seek how often 3 is contained in 17: the answer is 5; but multiplying the divisor thereby, the product is 1925, greater than the dividual: wherefore I say it will bear 4, an unit less,

```
385) 1183653 (30
     1155
     ----
      286
```

```
385) 1183653 (307
     1155
     ----
      2865
      2695
      ----
       170
```

```
385) 1183653 (3074
     1155
     ----
      2865
      2695
      ----
       1703
       1540
       ----
       (163)
```

and

and by it I multiply the divisor 385; and the product is 1540, which is less than the dividual; and therefore I put 4 in the quotient, and subtract the said product from the dividual, and there remaineth 163. And thus the work is finished; and I find that 1183653 being divided by 385, or into 385 equal shares or parts, the quotient or one of those parts, is 3074; and besides there is 163 remaining.

And thus the learner being well versed in the method of the foregoing examples, he may be sufficiently qualified for the dividing of any greater sum or number, into as many parts as he pleaseth; that is, he may understand the method of dividing by a divisor which consisteth of 4, or 5, or 6, or any greater number of places; the method being the same with the foregoing examples in every respect.

Other examples in division.

27986) 835684790 (29860
55972
275964
251874
240907
223888
170199
167916
Remains (22830)

196374) 473986018 (2413
392748
812380
785496
268841
196374
724678
589122
Remains (135556)

So if you divide 47386473 by 58736, you will find the quotient to be 806, and 45257 will remain after the work is ended.

In like manner, if you would divide 3846739:04 by 483064, the quotient will be 7963, and the remainder after division will be 100572.

Compendiums in division.

1. IF any given number be to be divided by another number that hath ciphers annexed on the right side thereof; omitting the ciphers, you may cut off so many figures from the right hand of the dividend, as there are ciphers before the divisor, and let the remaining numbers in the dividend be divided by the remaining number or numbers of the divisor; observing this caution, that if after your division is ended any thing remain, you are to annex thereto the number or numbers that were cut off from the dividend *; and such new found number shall be the remainder. As for example: Let it be required to divide 46658 by 400. Now, because there are two ciphers before the divisor, I cut off as many figures from before the dividend, *viz.* 58; so that then there will remain only 466 to be divided by 4, and the quotient will be 116, and there will remain 2; to which I annex the two figures (58) which were cut off from the dividend, and it makes 258 for the true remainder: so that I conclude 46658 being divided by 400, the quotient will be 116, and 258 remains after the work is ended; as by the work in the margin.

```
4|00) 466|58 (116
      4
      —
       6
       4
       —
       26
       24
       ——
      (258)
```

2. And hence it followeth, † that if the divisor be 1, or an unit with ciphers annexed, you may cut off so many figures from the dividend, as there are ciphers in the divisor; and then the figure or figures that are on the left

* *Et si divisor adjunctos sibi habeat circulos ad dextram; omissis circulis, et abscissis totidem ultimis figuris dividendi, in numeris reliquis fiat divisio; in fine autem divisionis restituendi sunt tum omissi circuli tum figuræ abscissæ.* Ough. Clav. Matth. cap. 5. 3.

† *Divisurus quemcunque numerum per 10, aufer ex dextra parte unicam eamque primam figuram: reliquæ enim figuræ productum ostendunt; ablatum residuum, &c.* Gem. Fris. arith. pars 1.

hand will be the quotient, and those that are on the right hand will be the remainder after the division is ended. As, thus: If 45783 were to be divided by 10, I cut off the last figure (3) with a dash thus, (4578|3), and the work is done; and the quotient is 4578, (the number on the left hand of the dash), and the remainder is 3 (on the right hand). In like manner, if the same number 45783 were to be divided by 100, I cut off two figures from the end thus, (457|83); and the quotient is 457; and the remainder is 83. And if I were to divide the same by 1000, I cut off three figures from the end thus, (45|783); and the quotient is 45, and 783 the remainder, *&c.*

3. The general effect of division is contained in the definition of the same; that is, by having two unequal numbers given, to find a third number in such proportion to the dividend, as the divisor hath to unit, or 1. It also discovers what reason or proportion there is between numbers: so if you divide 12 by 4, it quotes 3; which shews the reason or proportion of 4 to 12 is triple.

The second effect is, by the superficial measure or content, and the length of any oblong, rectangular parallelogram, or square plane, known, to find out the breadth thereby; or contrariwise, by having the superficies and breadth of the said figure, to find out the length thereof: also, by having the solidity and length of a solid, to find the superficies of the base; *& contra.*

The third effect is, by the contents, reason, price, value, buying, selling, expenses, wages, exchange, interest, profit or loss of any number of things, (be it money, merchandise, or what else), to find out the contents, reason, price, value, buying, selling, expense, wages, exchange, interest, profit, or loss, of any one thing of like kind.

The fourth effect is, to aid, to compose, and to make other rules; but principally the rule of proportion, called the *golden rule*, or *rule of three*; and the reduction of monies, weights, and measures, of one denomination, into another. By it also fractions are abbreviated, by finding a common measurer unto the numerator and denominator, thereby discovering commensurable numbers.

If you divide the value of any certain quantity by the same

same quantity, the quotient discovers the rate or value of the integer. As if 8 yards of cloth cost 96 shillings; if you divide 96, the value or price of the given quantity, by 8, the quotient will be 12 s. which is the price or value of 1 of those yards; *& contra*.

If you divide the value or price of any unknown quantity by the value of the integer, it gives you in the quotient that unknown quantity whose price is thus divided. As, if 12 shilling were the value of 1 yard, I would know how many yards are worth 96 shillings. Here, if you divide 96, the price or value of the unknown quantity, by 12, the rate of the integer, or 1 yard, the quotient will be 8; which is the number of yards worth 96 shillings.

Some questions answered by division, may be these following.

Quest. 1. If 22 things cost 66 shillings, what will 1 such thing cost? *Facit* 3 shillings: for if you divide 66 by 22, the quotient is 3 for the answer. So if 36 yards or ells of any thing be bought or sold for 108 l. how much will 1 yard or ell be bought or sold for? *Facit* 3 l. for if you divide 108 l. by 36 yards, the quotient will be 3 l. the price of the integer.

```
7) 868 (124
   ...
   7
   --
   16
   14
   ---
    28
    28
   ---
   (0)
   ===
```

Quest. 2. If the expense, charges, or wages of 7 years amount to 868 l. what is the expense, charges, or wages of 1 year? *Facit* 124 l. for if you divide 868 (the wages of 7 years) by 7 (the number of years), the quotient will be 124 l. for the answer. See the work.

```
9) 144 (16 inches,
   ..
   9
   --
   54
   54
   --
   (0)
```

Quest. 3. If the content of one superficial foot be 144 inches, and the breadth of a board be 9 inches, how many inches of that board in length will make such a foot? *Facit* 16 inches: for by dividing 144 (the number of square inches

in

in a ſquare foot) by 9 (the inches in the breadth of the board), the quotient is 16 for the number of inches in length of that board to make a ſuperficial foot.

Queſt. 4. If the content of an acre of ground be 160 ſquare perches, and the length of a furlong (propounded) be 80 perches, how many perches will there go in breadth to make an acre? *Facit* 2 perches; for if you divide 160 (the number of perches in an acre) by 80 (the length of the furlong in perches), the quotient is 2 perches; and ſo many in breadth of that furlong will make an acre.

80) 160 (2 perches
160
———
(0)

Queſt. 5. If there be 893 men to be made up into a battle, the front conſiſting of 47 men, what number muſt there be in the file? *Facit* 19 deep in the file; for if you divide 893 (the number of men) by 47 (the number in the front), the quotient will be 19 in depth of the file. See the work in the margin.

47) 893 (19 deep in file.
47
———
423
423
———
(0)

Queſt. 6. There is a table, whoſe ſuperficial content is 72 feet, and the breadth of it at the end is 3 feet; now I demand what is the length of this table? *Facit* 24 feet long; for if you divide 72 (the content of the table in feet) by 3 (the breadth of it), the quotient is 24 feet for the length thereof, which was required. See the operation in the margin.

3) 72 (24
6
—
12
12
—
(0)

The proof of multiplication and diviſion.

Multiplication and diviſion interchangeably prove each other; for if you would prove a ſum in diviſion, whether the operation be right or no, multiply the quotient by the diviſor; and if any thing remain after the diviſion is ended, add it to the product, which product, if your ſum was rightly divided, will be equal to the dividend. And contrariwiſe, if you would prove a ſum in multiplication, divide

vide the product by the multiplier, and if the work was rightly performed, the quotient will be equal to the multiplicand. See the example where the work is done and undone. Let 7654 be given to be multiplied by 3242, the product will be 24814268, as by the work appeareth.

```
    7654
    3242
  ------
   15308
  30616
 15308
22962
--------
24814268
========
```

And then if you divide the said product 24814268 by 3242 the multiplier, the quotient will be 7654, equal to the given multiplicand.

```
3242) 24814268 (7654
      22694
      -----
       21202
       19452
       -----
        17506
        16210
        -----
         12968
         12968
         -----
          (0)
```

In like manner, (to prove a sum or number in division), if 24814268 were divided by 3242, the quotient will be found to be 7654. Then for proof, if you multiply 7654 the quotient, by 3242 the divisor, the product will amount to 24814268, equal to the dividend.

```
    7654
    3242
  ------
   15308
  30616
 15308
22962
--------
7654) 24814268 (3242
```

Or you may prove the last, or any other example in multiplication, thus, *viz.* Divide the product by the multiplicand, and the quotient will be equal to the multiplier. See the work.

```
      22962
      -----
       18522
       15308
       -----
        32146
        30616
        -----
         15308
         15308
         -----
          (0)
```

From

From whence there ariseth this corollary, That any operation in division may be proved by division: for if after your division is ended, you divide the dividend by the quotient, the new quotient thence arising will be equal to the divisor of the first operation; for trial whereof, let the last example be again repeated.

```
3242) 24814268 (7654
      ....
      22694
      -----
       21202
       19452
       -----
        17506
        16210
        -----
         12968
         12968
         -----
          (0)
```

For proof whereof, divide again 24814268 by the quotient 7654, and the quotient hence will be equal to the first divisor 3242. See the work.

```
7654) 24814268 (3242
      ....
      22962
      -----
       18522
       15308
       -----
        32146
        30616
        -----
         15308
         15308
         -----
          (0)
```

But in proving division by division, the learner is to observe this following caution, That if after his division is ended, there be any remainder; before you go about to prove your work, subtract that remainder out of your dividend, and then work as in the following example, where it is required to divide 43876 by 765, the quotient here is 57, and the remainder is 271. See the work in the margin.

```
765) 43876 (57
     ..
     3825
     ----
      5626
      5355
      ----
      (271)
```

Now, to prove this work, subtract the remainder 271 out of the dividend 43876, and there remaineth 43605 for a new dividend to be divided by the former quotient 57, and the quotient thence arising is 765, equal to the

the given divisor, which proveth the operation to be right.

```
         43876
           271
      ---------
57) 43605 (765
       ...
       399
      ------
       370
       342
      ------
       285
       285
      ------
       (0)
```

Thus have we gone through the four species of arithmetic, *viz.* addition, subtraction, multiplication, and division, upon which the following rules, and all other operations whatsoever that are possible to be wrought by numbers, have their immediate dependence, and by them are resolved. Therefore before the learner make a farther step in this art, let him be well acquainted with what hath been delivered in the foregoing chapters *.

C H A P. VIII.

Of Reduction.

1. REduction is that which brings together two or more numbers of different denominations into one denomination; or it serveth to change or alter numbers, money, weight, measure, or time, from one denomination to another; and likewise to abridge fractions to the lowest terms; all which it doth so precisely, that the first proportion remaineth without the least jot of error or wrong committed: so that it belongeth as well to fractions as integers; of which in its proper place. Reduction is

* *Hae sunt igitur quatuor illae species arithmetices per quas omnia quaecunque deinceps dicenda sunt, vel quae per numeros fieri possibile est absolvuntur; quare eas quisquis es ante omnia perdiscet.* Gem. Fris. arithm. par. 1.

generally

generally performed either by multiplication or division. From whence we may gather, That,

2. Reduction is either descending or ascending.

3. Reduction descending is, when it is required to reduce a sum or number of a greater denomination into a lesser, which number, when it is so reduced, shall be equal in value to the number first given in the greater denomination: as if it were required to know how many shillings, pence, or farthings, are equal in value to an hundred pounds? or, how many ounces are contained in 45 hundred weight? or, how many days, hours, or minutes, there are in 240 years? *&c.* And this kind of reduction is generally performed by multiplication.

4. Reduction ascending is, when it is required to reduce or bring a sum or number of a smaller denomination into a greater, which shall be equivalent to the given number: as suppose it were required to find out how many pence, shillings, or pounds, are equal in value to 43785 farthings? or, how many hundreds are equal to (or in) 3748 pounds, *&c.* And this kind of reduction is always performed by division.

5. When any sum or number is given to be reduced into another denomination, you are to consider whether it ought to be resolved by the rule descending or ascending, *viz.* by multiplication, or division. If it be to be performed by multiplication, consider how many parts of the denomination into which you would reduce it are contained in an unit or integer of the given number, and multiply the said given number thereby, and the product thereof will be the answer to the question. As, if the question were, In 38 pounds how many shillings?

Here I consider, that in 1 pound are 20 shillings, and that the number of shillings in 38 pounds, will be 20 times 38; wherefore I multiply 38 l. by 20, and the product is 760, and so many shillings are contained in 38 pounds, as in the margin.	38 20 ——— 760

But when there is a denomination or denominations between the number given and the number required, you may, if you please, reduce it into the next inferior denomination, and then into the next lower than that, *&c.*

until you have brought it into the denomination required. As for example: Let it be demanded, In 132 pounds how many farthings? Firſt, I multiply 132 (the number of pounds given) by 20 to bring it into ſhillings, and it makes 2640 ſhillings. Then do I multiply the ſhillings 2640 by 12, to bring them into pence, and it produceth 31680, and ſo many pence are contained in 2640 ſhillings, or 132 pounds. Then do I multiply the pence, *viz.* 31680 by 4 to bring them into farthings, (becauſe 4 farthings is a penny), and I find the product thereof to be 126720, and ſo many farthings are equal in value to 132 pounds. The work is manifeſt in the margin.

```
   132 pounds
    20
  ----
  2640 ſhillings
    12
  ----
   528
  264
 -----
 31680 pence
     4
 -----
126720 farth.
```

6. And if the number propounded to be reduced is to be divided, or wrought by the rule aſcending, conſider how many of the given numbers are equal to an unit or integer in that denomination to which you would reduce your given number, and make that your diviſor, and the given number your dividend; and the quotient thence ariſing will be the number ſought or required. As for example: Let it be required to reduce 2640 ſhillings into pounds. Here I conſider that 20 ſhillings are equal to 1 pound; wherefore I divide 2640 (the given number) by 20, and the quotient is 132, and ſo many pounds are contained in 2640 ſhillings. In reduction deſcending and aſcending, the learner is adviſed to take particular notice of the tables delivered in the ſecond chapter of this book, where he may be informed what multipliers or diviſors to make uſe of in the reducing of any number to any other denomination whatſoever, eſpecially Engliſh monies, weights, meaſures, time, and motion.

```
            l.
2|0 ) 264|0 ( 132
      . . .
      2
      —
      6
      6
      —
       4
       4
       —
      (0).
```

But

But in this place it is not convenient to meddle with foreign coins, weights, or measures.

But if in reduction ascending it happen that there is a denomination or denominations between the number given and the number required, then you may reduce your number given into the next superior denomination; and when it is so reduced, bring it into the next above that; and so on, until you have brought it into the denomination required. As for example:

Let it be demanded, In 126720 farthings how many pounds? First I divide my given number (being farthings) by 4, to bring them into pence, (because 4 farthings make one penny); and there are 31680 pence. Then I divide 31680 pence by 12, and the quotient giveth 2640 shillings. And then I divide 2640 shillings by 20, and the quotient giveth 132 pounds; which are equal in value to 126720 farthings. See the whole work as it followeth.

```
                  12)      2|0)     l.
      4) 126720 (31680 (264|0 (132
         .....   ....   ..
         12      24     2
         --      --     --
          6      76     6
          4      72     6
         --      --     --
         27      48     4
         24      48     4
         --      --     --
          32     (0)    (0)
          32
          --
          (0)
```

7. When the number given to be reduced, consisteth of divers denominations, as pounds, shillings, pence, and farthings; or of hundreds, quarters, pounds, and ounces, *&c.* then you are to reduce the highest (or greatest) denomination into the next inferior, and add thereunto the number standing in that denomination, which your greatest or highest number is reduced to. Then reduce that sum into the next inferior denomination, adding thereto the number standing in that denomination. Do so until you have brought the number given into

into the denomination proposed. As, if it were required to reduce 48 l. 13 s. 10 d. into pence: first I bring 48 l. into shillings, by multiplying it by 20, and the product is 960 shillings; to which I add the 13 shillings, and they make 973. Then I multiply 973 by 12, to bring the shillings into pence, and they make 11676 pence; to which I add the 10 pence, and they make 11686 pence for the answer. See the work done.

	l.	s.	d.
	48	— 13 —	10
	20		
	960 shillings		
Add	13		
Sum	973 shillings		
	12		
	1946		
	973		
	11676 pence		
Add	10		
Sum	11686 pence		

8. If in reduction ascending, after division is ended, any thing remain, such remainder is of the same denomination with the dividend.

Example. In 4783 farthings, I demand how many pounds? View the following operation.

First, I divide the given number of farthings (*viz.* 4783) by 4, to bring them into pence, and the quotient is 1195 pence; and there remaineth 3 after the work of division is ended, which is 3 farthings.

Again, I divide 1195 pence (the said quotient) by 12, to reduce them into shillings, and the quotient is 99 shillings; and there is a remainder of 7, which is 7 pence.

	12)	2|0)	
4) 4783	(1195	(9|9	(4 pounds
4	108	8	
7	115	(19)	shillings
4	108		
38	Rem. (7) pence		
36			
23	*Facit*	l. s. d. q.	4—19—7—3
20			
Rem. (3) farthings			

And then divide 99 shillings (the last quotient) by 20,

20, to bring it into pounds, and the quotient is 4 l. and there remaineth 19 ſhillings: ſo that I conclude that in 4783 (the propoſed number of farthings) there is 4 pounds, 19 ſhillings, 7 pence, 3 farthings.

More examples in reduction of coin.

Queſt. 1. In 438 l. how many ſhillings? *Facit* 8760 ſhillings; for multiplying 438 by 20, the product amounteth to ſo much. See the work.

438 pounds
20
Facit 8760 ſhillings

Queſt. 2. In 467 l. how many pence? Firſt multiply the given number of pounds (467) by 20, to bring it into ſhillings, and it makes 9340 ſhillings; then multiply the ſhillings by 12, and it produceth 112080 pence: thus.

467 pounds
20
9340 ſhill.
12
1868
934
Facit 112080 pence

Or it may be reſolved thus, *viz.* Multiply the given number of pounds (467) by (240), the number of pence in a pound, and the product is the ſame, *viz.* 112080 pence; as by the operation appeareth.

467 pounds
240
1868
934
Facit 112080 pence

Queſt. 3. In 5673 l. how many farthings? Firſt multiply the given number by 20, to bring it into ſhillings, and it produceth 113460 ſhillings; then multiply that product by 12, to bring it into pence, and it produceth 1361520 pence; then, laſtly, multiply the pence by 4, and it produceth 5446080 farthings. See the operation.

5673 pounds
20
113460 ſhillings
12
22692
11346
1361520 pence
4
Facit 5446080 farthings

Or

Or this question might have been thus resolved, *viz.* Multiply 5673 (the given number of pounds) by 960 (the number of farthings in a pound), and it produceth the same effect; as you may see by the work.

5673	pounds	20	shillings
960			
34038		240	pence
51057		4	
Facit 5446080	farthings	960	farthings

Otherwise thus: First, bring the given number 5673 l. into shillings, and multiply the shillings by 48, the number of farthings in a shilling, and the same effect is thereby likewise produced, *viz.*

5673	pounds	12	pence
20		4	
113460	shillings	48	farthings
48			
90768			
45384			
Facit 5446080	farth.		

These various ways of operation are expressed to inform the judgment of the learner with the reason of the rule. More ways may be shewn, but these are sufficient even for the meanest capacities.

Quest.

Quest. 4. In 458 *l.* 16 *s.* 7 *d.* 3 qrs. how many farthings? To resolve this question, consider the seventh rule of this chapter, and work as you are there directed, and you will find the aforesaid given numbers to amount to 440479 farthings, *viz.*

	l.	*s.*	*d.*	*q.*
	458	16	7	3
	20			
	9160			
Add	16 shillings			
Sum	9176 shillings			
	12			
	18352			
	9176			
	110112			
Add	7 pence			
Sum	110119 pence			
	4			
	440476			
Add	3 farthings			
Sum	440479 farthings			

This last question, or any other of this kind, *viz.* where the number given to be reduced consisteth of several denominations, may be more concisely resolved thus, *viz.* When you multiply the pounds by 20, to bring them into shillings, to the product of the first figure add the figure standing in the place of units in the denomination of shillings: but because the first figure in the multiplier is 0, I say, 0 times 8 is nothing, but 6 is 6, which I put down for the first figure in the product. Then, because the multiplier is 0, I go on no further with it; for if I should, the whole product would be 0; but proceed. And when I come to

	l.	*s.*	*d.*	*q.*
	458	16	7	3
	20			
	9176 shillings			
	12			
	18359			
	9176			
	110119 pence			
	4			
Facit	440479 farthings			

to multiply by the ſecond figure in the multiplier, to the product of it I add the figure ſtanding in the place of tens in the denomination of ſhillings, which is 1; ſaying, 2 times 8 is 16 and (the ſaid figure) 1 is 17. Then I ſet down 7, and carry the unit to the product of the next figure, as is directed in the fifth rule of the ſixth chapter foregoing, and I finiſh the work. So that now you may have the whole product and ſum of ſhillings at one operation, which is the ſame as before. And when you multiply the ſhillings by 12, to bring them into pence, after the ſame manner, add to the product the number ſtanding in the denomination of pence; and ſo when you multipy the pence by 4, to bring them into farthings, add to the product the number ſtanding under the denomination of farthings. See the laſt queſtion thus wrought on the margin of the preceding page.

After the method laſt preſcribed (which, if rightly conſidered, differeth not any thing from the ſeventh rule of this chapter) are all the following examples that are of the ſame nature wrought and reſolved.

Queſt. 5. In 4375866 farthings, I demand how many pounds, ſhillings, pence, and farthings?

To reſolve this queſtion; firſt, I divide the given number of farthings by 4, and the quotient is 1093966 pence; and there remaineth 2 after the diviſion is ended; which, by the eighth rule foregoing, is 2 farthings. Then I divide 1093966 pence by 12, and the quotient is 91163 ſhillings; and there remaineth 10 after diviſion; which, by the ſaid eighth rule, is ſo many pence, *viz.* 10 d. Then I divide 91163 ſhillings by 20, and the quotient is 4558 l. and there remaineth 3 ſhillings. So the work is finiſhed, and I find that in 4375866 farthings, there are 4558 l. 3 s. 10 d. 2 qrs. See the operation following.

12)

```
          12)          2|0         l.
4) 4375866 (1093966 (9116|3 (4558
   .......   .....     ....
   4         108        8
   --        ---        --
   37         13        11
   36         12        10
   --         --        --
    15        19        11
    12        12        10
    --        --        --
     38       76        16
     36       72        16
     --       --        --
      26      46       (3) s.
      24      36
      --      --
       26     (10) d.
       24
       --
      (2) qrs.
```

l. *s.* *d.* *qrs.*
Facit 4558——3——10——2

Quest. 6. In 4386 l. I demand how many groats?

To resolve this question, I reduce the given number of pounds into shillings, and they are 87720 shillings. Now I consider, that in a shilling are 3 groats; therefore I multiply the shillings by 3, and it produceth 263160 groats. See the work.

```
        4386 pounds
          20
       ------
       87720 shillings
           3
       ------
Facit 263160 groats
```

This question might have been otherwise resolved thus, *viz.* Considering that in a pound (or 20 shillings) there are 3 times 20 groats, which makes 60; therefore I multiply the number of pounds given by 60, and it produceth the same effect at one operation. See the margin.

```
        4386 pounds
          60 groats in 20 s.
      ------
Facit 263160 groats in 4386 l.
```

Quest. 7. In 43758 threepences, I desire to know how many pounds?

To resolve this, and many such like questions; first, I divide my given number of threepences by 4, because 4 threepences are a shilling, and the quotient is 10939 shillings; and there remaineth 2 after the division is ended, which is 2 threepences, (by the eighth rule of this chapter), which are equal in value to 6 d. Then I divide 10939 shillings by 20, and the quotient giveth 546 l. and 19 s. remains: so that I conclude, in 43758 pieces of three pence *per* piece, there are 546 l. 19 s. 6 d. as by the work appeareth.

```
              2|0         l.   s.  d.
4) 43758 (1093|9 (546—19—6
   ....    ...
    4      10
   ———    ———
    37      9
    36      8
   ———    ———
     15     13
     12     12
    ———   ———
      38    19 shillings
      36
     ———
     (2) threepences, or 6 d.
```

This question might have been otherwise resolved thus, *viz.* First, multiply the given number of threepences (43758) by 3, the number of pence in threepence, and the product (*viz.* 131274) is the number of pence equal to the given number of threepences; which number of pence may be brought into pounds, by dividing by 12 and by 20, and the quotient you will find to be equal to the former work, *viz.* 546 l. 19 s. 6 d.

```
      43758
          3
      ------     2|0          l.   s.  d.
12)  131274   (1093|9    (546—19—6
      ...         ...
      12          10
      ---         ---
      112          9
      108          8
      ---         ---
        47        13
        36        12
        ---       ---
        114 Rem. (19) shillings
        108
        ---
Rem.   (6)
```

Or thus: Divide the given number of threepences by the number of threepences in a pound or 20 shillings, (which you will find to be 80, if you multiply 20 s. by 4, the number of threepences in a shilling), and you will find the quote to be 546 l. as before, and a remainder of 78 threepences; and if you divide those 78 threepences by 4, (because there are 4 threepences in a shilling), you will find the quote to be 19 s. and 2 threepences remain, which are equal to 6 d. which is the same that was before found.

```
8|0)  4375|8  (546—19—6        20
       ...                      4
       40                      ---
       ---                     80
        37
        32
        ---
         55
         48
         ---
    4)  78  (19 s.
        ..
         4
        ---
         38
         36
         ---
        (2) threepences, or 6 d.
```

Quest. 8. In 4785 l. 13 s. how many pieces of $13\frac{1}{2}$ d. *per* piece?

This question cannot be resolved by reduction descending or ascending absolutely, (because $13\frac{1}{2}$ d. is no even part of a pound), but rather by them both jointly, *viz.* by multiplication and division. For if you bring the number given into halfpence, and divide the halfpence by the halfpence in $13\frac{1}{2}$, *viz.* 27, the quotient will be the answer: So having brought 4785 l. 13 s. into halfpence, I find it makes 2297112; which I divide by 27, (because there are so many halfpence in $13\frac{1}{2}$ d.), and the quote gives 85078 pieces of $13\frac{1}{2}$ d. and 6 halfpence remain over and above. Observe the work following.

l. *s.*		*d.*
4785—13		$13\frac{1}{2}$ d.
20		2
95713	shillings	27 halpence
24	halfpence in a shilling.	
382852		
191426		
2297112	halfpence in the given number	

27) 2297112 (85078 pieces of $13\frac{1}{2}$ d.

216
137
135
211
189
222
216

Rem. (6) halfpence

It would have produced the same answer, if you had reduced your given number into farthings, and divided by the farthings in $13\frac{1}{2}$ d. *viz.* 54; (for always the dividend

dend and the divisor must be of one denomination); and then you would have had a remainder of 12 farthings, which are equal in value to the former remainder of 6 halfpence; as you may prove at your leisure.

Quest. 9. In 540 dollars, at 4 s. 4 d. *per* dollar, how many pounds Sterling?

First, bring your given number of dollars into pence, and then your pence into pounds, according to the former directions. Thus, in 4 s. 4 d. (*viz.* a dollar) you will find 52 pence; by which multiply 540 dollars, and it produceth 28080 pence; which if you divide by 240, (the pence in one pound), the quotient will give you 117 l. which are equal in value to 540 dollars, at 4 s. 4 d. *per* dollar.

```
                      s.  d.
       540            4—4
        52             12
      ----           ----
       108             52
       270
      ----
24|0) 2808|0 (117 l.
       24
      ----
        40
        24
      ----
        168
        168
      ----
        (0)
```

The foregoing question might have been otherwise wrought thus; *viz.* Multiply (540) your given number of dollars, by 13, the number of groats in a dollar (or 4 s. 4 d.), and it produceth 7020 groats; which divide by 60 (the groats in 1 pound or 20 shillings), and the quote is 117 l. as before. See the work.

```
                      s.  d.
       540            4—4
        13              3
      ----           ----
       162             13.
       54
      ----
6|0) 702|0 (117 l.
      6
     ---
      10
       6
     ---
       42
       42
     ---
      (0)
```

Quest. 10. In 547386 pieces of $4\frac{1}{2}$ d. *per* piece, I demand how many pounds, shillings, and pence?

First, bring your given number of four pence halfpenny all into halfpence, which you will do if you multiply by 9, the number of halfpence in $4\frac{1}{2}$ d. and the product is 4926474 halfpence; which are brought into pounds, if you divide them by 24 (the halfpence in a shilling), and 20 (the shillings in a pound). It makes 10263 l. 9 s. 9 d. as by the work.

```
                                     d.
                                     4½
   547386                            2
        9                            ——
  ———————      2|0)       l.         9 halfpence
24)4926474  (20526|9  (10263
   ••••••    •••••
   48         2
   ——        ——
   126        5
   120        4
   ———       ——
    64       12                      l.   s.  d.
    48       12              Facit 10263—9—9
   ———       ——
    167       6
    144       6
    ———      ——
     234 Rem. (9) shillings
     216
     ———
Rem. (18) halfpence, or 9 d.
```

Quest. 11. In 4386 l. I demand how many pieces of 6 d. of 4 d. and of 2 d. of each an equal number? that is to say, What number of sixpences, groats, and twopences, will make up 4386 l. and the number of each equal?

The way to resolve questions of this nature, is to add the several pieces into which the given number is to be brought, into one sum, and to reduce the given number into the same denomination with their sum, and to divide the said given number, so reduced, by the said sum,

ſum, and the quotient will give you the exact number of each piece. And after the ſame method will we proceed to reſolve the preſent queſtion, *viz.*

```
   4386  pounds                         d.
    240  pence                          6
   ----                                 4
  17544                                 2
  8772                                  —
  -------                       Sum 12 pence
12) 1052640 (87720
    .....
   96
   --
    92
    84
    --
     86
     84                         d. d. d.
     --              Facit 87720 pieces of 6—4—2
      24
      24
      --
      (0)
```

So that I conclude by the operation, that 87720 ſixpences, 87720 groats, and 87720 twopences, are juſt as much as (or equal to) 4386 l. : or if you admit of 5 s. to be thus divided, it is equal to 5 ſixpences, and 5 fourpences or groats, and 5 twopences. "For if two right "lines (or two numbers) be given, and one of them be di-"vided into as many parts or ſegments as you pleaſe, the "rectangle (or product) comprehended under the two "whole right lines (or numbers given) ſhall be equal to "all the rectangles (or products) contained under the "whole line (or number), and the ſeveral ſegments (or "parts) into which the other line (or number) is divided. "*Eucl.* 2. 1."

Another queſtion of the ſame nature with the laſt, may be this following, *viz.*

Queſt. 12. A merchant is deſirous to change 148 l. into pieces of $13\frac{1}{2}$ d. of 12 d. of 9 d. of 6 d. and of 4 d. and he

he will have of each sort an equal number of pieces; I desire to know the number?

Do as you are taught in the last question, *viz* Add the several pieces together, and reduce the sum into halfpence; then reduce the sum to be changed, *viz.* 148 l. into the same denomination, and divide the greater by the lesser, and in the quotient you will find the answer, *viz.* 798 is the number of each of the pieces required, and 18 remaineth, which is 18 halfpence, by the eighth rule of this chapter. See the work as followeth.

```
 l.                                        d.
 148                                       13½
  240 pence in a pound.                    12
 ----                                       9
  592                                       6
 296                                      ---
 -----                                      4
 35520 pence in 148 l.                    ---
     2                                 Sum 44½
 -----                                      2
 71040 halfpence                          ---
                                           89 halfpence
      89) 71040 (798 pieces of each sort
            ...
           623
           ---
           874
           801
           ---
            730
            712
            ---
     Rem (18) halfpence.
```

The truth of the two foregoing operations will thus be proved, *viz.* Multiply the answer by the parts or pieces into which the given number was reduced; and having added the several products together, if their sum be equal to the given number, the answer is right; otherwise not.

So the answer to the 11th question was 87720; which is proved as followeth, *viz.*

87720

			l.
87720	ſixpences make	—	2193
	fourpences make	—	1462
	twopences make	—	731

The total ſum of them 4386 which was the ſum given to be changed.

The anſwer to the 12th queſtion was 798, and 18 halfpence remained after the work was ended. Now, the truth of the work may be proved as the former was, *viz.*

		d.		*l.*	*s.*	*d.*
789	pieces of	$13\frac{1}{2}$	make	44	17	9
	pieces of	12	make	39	18	0
	pieces of	9	make	29	18	6
	pieces of	6	make	19	19	0
	pieces of	4	make	13	6	0
and 18 halfpence, or 9 d. remains				0	0	9
The total ſum of them				148	0	0

which total ſum is equal to the number that was firſt given to be changed; and therefore the operation was rightly performed.

Reduction of Troy weight.

We now come to give the learner ſome examples in Troy weight; wherein we ſhall be brief, having given ſo large a taſte of reduction in the former examples of coin. And now the learner muſt be mindful of the table of Troy weight delivered in the ſecond chapter of this book.

Queſt. 13. In 482 lb. 7 oz. 13 pw. 21 gr. how many grains?

Multiply by 12, by 20, and by 24, taking in the figures ſtanding in the ſeveral denominations, according to the directions given in the ſeventh rule of this chapter, and you will find the product to be 2780013 grains, which is the number required, or anſwer to the queſtion. See the whole work as followeth.

l.	*oz.*	*pw.*	*gr.*
482	7	13	21

12
———
971
482
———
5791 ounces
20
———
115833 penny-weight.
24
———
463333
231668
———
Facit 2780013 grains.

Quest. 14. In 2780013 grains, I demand how many pounds, ounces, penny-weights, and grains?

This is but the foregoing question inverted, and is resolved by dividing by 24, by 20, and by 12; and the answer is 482 lb. 7 oz. 13 pw. 21 gr.

24) 2780013	2|0 (115833|3	12) (5791	*lb.* (482
24	10	48	
38	15	99	
24	14	96	
140	18	31	
120	18	24	
200	3	Rem. (7) ounces	
192	2		
81	Rem. (13) penny-weight.		
72			
93			
72			
Remains (21) grains			

	lb.	*oz.*	*pw.*	*gr.*
Facit	482	7	13	21

Quest.

Quest. 15. A merchant ſent to a goldſmith 16 ingots of ſilver, each containing in weight 2 lb. 4 oz. and ordered it to be made into bowls of 2 lb. 8 oz. *per* bowl, and tankards of 1 lb. 6 oz. *per* piece, and ſalts of 10 oz. 10 pw. *per* ſalt, and ſpoons of 1 oz. 18 pw. *per* ſpoon, and of each an equal number; I deſire to know how many of each ſort he muſt make?

This queſtion is of the ſame nature with the 11th and 12th queſtions foregoing, and may be anſwered after the ſame method, *viz.* Firſt, add the weight of the ſeveral veſſels into which the ſilver is to be made, into one ſum, and reduce them to one denomination, and they make 1248 penny-weights; then reduce the weight of the ingot into the ſame denomination, *viz.* penny weights, and it makes 560 penny-weights; and multiply them by the number of ingots, *viz.* 16, and the product will give you the weight of the 16 ingots, *viz.* 8960 penny weights; then divide the product by the weight of the veſſels, *viz.* 1248, and the quotient giveth you the anſwer to the queſtion, *viz.* 7, and 224 pw. remaineth over and above.

```
lb.     oz.                     lb.   oz.   pw.
 2 ——— 4                         2 —— 8 ——— 0
12                               1 —— 6 ——— 0
——                               0 —— 10 —— 10
28                               0 —— 1 ——— 18
20                              ——————————————
——                          Sum  5 —— 2 ——— 8
560 penny-weights               12
16  ingots                      ——
———                             62 oz.
336                             20
56                             ———
———                            1248 pw.
1248) 8960 (7 veſſels of each ſort
      8736
      ————
Rem. (224) penny-weights.
```

The

The proof of the work is as followeth.

		lb.	oz.	pw.		lb.	oz.	pw.
7	bowls of	2—	8—	0	*per* bowl is	18—	8—	0
	tank. of	1—	6—	0	*per* tank. is	10—	6—	0
	ſalts of	0—	10—	10	*per* ſalt is	6—	1—	10
	ſpoons of	0—	1—	18	*per* ſpoon is	1—	1—	6
	224 penny-weight remaining					0—	11—	4
					Total ſum	37—	4—	0

So that you ſee the ſum of the weight of all the veſſels, together with the remainder, is 37 lb. 4 oz. which is equal to the weight of the 16 ingots delivered. For if 37 lb. 4 oz. be reduced to penny-weights, it makes 8960.

Reduction of Avoirdupois weight.

In reducing Avoirdupois weight, the learner muſt have recourſe to the table of Avoirdupois weight delivered in the ſecond chapter foregoing.

Queſt. 16. In 47 C. 1 qr. 20 lb. how many ounces? Multiply by 4, by 28, and by 16; and the laſt product will be the anſwer, *viz.* 84992 ounces.

```
C.  qr. lb.
47—1—20
 4
-------
189 quarters
 28
-------
1512
380
-------
5312 lb.
 16
-------
31872
5312
-------
Facit 84992 ounces
```

Queſt. 17. In 84992 ounces, I demand how many C. qrs. lb. and oz.?

This is the foregoing queſtion inverted; and will be reſolved if you divide by 16, by 28, and by 4; and the anſwer

answer is 47 C. 1 q^r. 20 lb. equal to the given number in the foregoing question.

```
                28)    4)    C.  qr.  lb.  oz.
16) 84992 (5312 (189 (47—1—20—0
    ....     ...    ..
     80       28     16
    ----     ----   ----
      49     251     29
      48     224     28
    ----     ----   ----
       19     272   (1) qr.
       16     252
     ----    ----
        32   (20) pounds
        32
      ----
       (0)
```

Quest. 18. A chapman buyeth of a grocer 4 C. 1 qr. 14 lb. of pepper, and ordereth it to be made up into parcels of 14 lb. of 12 lb. of 8 lb. of 6 lb. and of 2 lb. and of each parcel an equal number: now I would know the number of each parcel?

This example is of the same nature with the 11th, 12th, and 15th questions foregoing; and after the same manner is resolved. See the operation as followeth.

```
       C.   qr.   lb.            lb.
       4——1——14                  14
       4                         12
      ---                         8
      17                          6
      28                          2
     ----                        ---
     140                         42 lb.
      35
     ----
 42) 490 (11
     42
    ----
      70                Facit 11 parcels of each.
      42
    ----
Rem. (28) pounds
```

Reduction of liquid measure.

Quest. 19. In 45 tun of wine, how many gallons? Multiply by 4, and by 63; the product is 11340 gallons for the answer.

```
   45
    4
  ---
  180
   63
  ---
   54
 108
 ----
Facit 11340 gallons
```

Quest. 20. In 34 rundlets of wine, each containing 18 gallons, I demand how many hogsheads?

First, find how many gallons are in the 34 rundlets, which you may do if you multiply 34 by 18, the content of a rundlet, and the product is 612 gallons; which you may reduce into hogsheads, if you divide them by 63, and the quote will be 9 hogsheads and 45 gallons. See the work.

```
      34
      18
     ---
     272
      34
     ---
63) 612 (9 hhds
    567
    ---
Remains (45) gallons
Facit 9 hhds 45 gall.
```

Quest. 21. In 12 tuns, how many rundlets of 14 gallons *per* rundlet?

Reduce your tuns into gallons, and divide them by 14, the gallons in a rundlet, and the quotient (216) is your answer. See the work in the margin..

```
      12
       4
      --
      48
      63
     ---
     144
    288
    ----
14) 3024 (216 rundlets
    28
    --
     22
     14
     --
      84
      84 Fa. 216 run.
      --
      (0)
```

Reduction of long measure.

Quest. 22. I demand how many furlongs, poles, inches, and barley-corns will reach from London to York, it being accounted 151 miles?

151	miles
8	furlongs in a mile
1208	furlongs
40	poles in a furlong
48320	poles
11	half-yards in a pole
4832	
4832	
531520	half-yards
18	inches in half a yard
425216	
53152	
9567360	inches
3	barley-corns in an inch
Facit 28702080	barley-corns in 151 miles.

Quest. 23. The circumference of the earth (as all other circles are) is divided into 360 degrees, and each degree into 60 minutes, which, upon the superficies of the earth, are equal to 60 miles; now, I demand how many miles, furlongs, perches, yards, feet, and barley-corns, will reach round the globe of the earth?

360	degrees
60	minutes or miles in a degree
21600	miles about the earth
8	furlongs in a mile
172800	furlongs about the earth
40	perches in a furlong
6912000	poles or perches about the earth
11	half-yards in a perch
6912	
6912	
2)76032000	half-yards about the earth
(38016000	yards, *viz.* the half-yards divided by 2
3	
114048000	feet about the earth
12	inches in a foot
228096	
114048	
1368576000	inches about the earth
3	barley-corns in an inch
Facit 4105728000	barley corns.

And so many will reach round the world, the whole being 21600 miles. So that if any person were to go round, and go 15 miles every day, he would go the whole circumference in 1440 days, which is 3 years, 11 months, and 15 days.

Reduction of time.

Quest. 24. In 28 years, 24 weeks, 4 days, 16 hours, 30 minutes, how many minutes?

Years.	*weeks.*	*days.*	*hours.*	*minutes.*
28	24	4	16	30

52 weeks in a year

60
142

1480 weeks
7

10364 days
24

41462
20729

248752 hours
60

14925150 minutes

Note, That in resolving the last question after the method expressed, there are lost in every year 30 hours; for the year consisteth of 365 days and 6 hours; but by multiplying the years by 52 weeks, which is 364 days, you lose 1 day and 6 hours every year. Wherefore, to find an exact answer, bring the odd weeks, days, and hours, into hours, and then multiply the years by the number of hours in a year, *viz.* 8766, and to the product add the hours contained in the odd time, and you have the exact time in hours; which bring into minutes, as before. See the last question thus resolved.

	Days. hours.	*Weeks. days. hours.*
28	365—6	24—4—16
8766	24	7
172	1466	172
172	730	24
197	8766 hours in a year	694
228		345
249592 hours		4144 hours
60		

14975520 min. in 28 years and 4144 hours.

So you see that according to the method first used to resolve this question, the hours contained in the given time, are 248752; but according to the last, best, or truest method, they are 249592, which exceeds the former by 840 hours.

But for most occasions it will be sufficient to multiply the given years by 365; and to the product add the days in the odd time, if there be any, and then there will be only a loss of 6 hours in every year; which may be supplied by taking a fourth part of the given years, and adding it to the contained days, and you have your desire.

Quest. 25. In 438657540 minutes, how many years? *Facit* 834 years, 4 days, 19 hours.

	8766	*Years. days. hours.*
6|0) 43865754|0	(7310959	(834—4—19
42	70128	
18	29815	
18	26298	
6	35179	
6	35064	
57	24) 115 (4 days	
54	96	
35	Rem. (19) hours	
30		
54		
54		
(0)		

Quest.

Quest. 26. I desire to know how many hours and minutes it is since the birth of our Saviour Jesus Christ to this present year, being accounted 1677 years?

This question is of the same nature with the 24th foregoing, and after the same manner is resolved, *viz.* Multiply the given number of years by 8766, and the product is 14700582 hours; and that by 60, and the product is 882034920 minutes. See the work.

1677	
8766	hours in a year.
10062	
10062	
11739	
13416	
14700582	hours in 1677 years.
60	
882034920	minutes in 1677 years.

Note, That as multiplication and division do interchangeably prove each other; so reduction descending and ascending prove each other by inverting the question, as the 13th and 14th; and likewise the 16th and 17th questions foregoing, by inversion, do interchangeably prove each other. The like may be performed for the proof of any question in reduction whatsoever.

Thus far have we discoursed concerning single arithmetic, whose nature and parts are defined in the second, eighth, ninth, and tenth definitions of the third chapter of this book. For although reduction is not reckoned or defined among the parts of single arithmetic; yet, considered abstractedly, it is the proper effect of multiplication and division. And as for the extraction of roots, (which ought to be handled in the next place as parts of single arithmetic), we shall omit it in this place; and refer the learner to Mr Cocker's *decimal arithmetic*, which is (with great care and pains) now published together with his *logarithmetical arithmetic*, shewing the genesis or fabric of the logarithms, and their general use in arithmetic, *&c.*; as also his *algebraical arithmetic*, containing the doctrine of composing and resolving an equation, with all other rules necessary for the understanding of that mysterious art, *&c.*

CHAP.

CHAP. IX.

Of comparative arithmetic, viz. *The relation of numbers one to another.*

1. COmparative arithmetic, is that which is wrought by numbers as they are considered to have relation one to another; and this consists either in quantity or in quality. *Vid. Boetius's arith. lib.* 1. *cap.* 21.

2. Relation of numbers in quantity, is the reference or respect that the numbers themselves have one to another; where the terms or numbers propounded are always two, the first called *the antecedent*, and the other *the consequent*. *Vid. Wing. arith. cap.* 34.

3. The relation of numbers in quantity consists in the differences, or in the rate or reason that is found betwixt the terms propounded; the differences of two numbers being the remainder found by subtraction; but the rate or reason betwixt two numbers is the quotient of the antecedent divided by the consequent. So 21 and 7 being given, the difference betwixt them will be found to be 14; but the rate or reason that is betwixt 21 and 7, will be found to be triple reason; for 21 divided by 7, quotes 3, the reason or rate. *Vid. Alsted. mathemat. lib.* 2. *cap.* 11. & 12.

4. The relation of numbers in quality, (otherwise called *proportion*), is the reference or respect that the reason of numbers have one to another: therefore the terms given ought to be more than two. Now, this proportion, or reason between numbers relating one to another, is either arithmetical, or geometrical. *Vid. Alsted. mathemat. lib.* 2. *cap.* 21.

5. Arithmetical proportion (by some called *progression*) is, when divers numbers differ one from another by equal reason; that is, have equal differences.

So this rank of numbers, 3, 5, 7, 9, 11, 13, 15, 17, differ by equal reason, *viz.* by 2; as you may prove.

6. In a rank of numbers that differ by arithmetical proportion, the sum of the first and last term being multiplied by half the number of terms, the product is the total sum of all the terms.

Or,

Or, if you multiply the number of the terms, by the half ſum of the firſt and laſt terms, the product is the total ſum of all the terms.

So, in the former progreſſion given, 3 and 17 is 20, which multiplied by 4, *viz.* half the number of terms, the product gives 80, the ſum of all the terms: or multiply 8 (the number of terms) by 10 (half the ſum of the firſt and the laſt term), the product gives 80, as before.

So alſo 21, 18, 15, 12, 9, 6, 3, being given, the ſum of all the terms will be found to be 84; for here the number of terms is 7, and the ſum of the firſt and laſt (*viz.* 21 and 3) is 24, half whereof (*viz.* 12) multiplied by 7, produceth 84, the ſum of the terms ſought.

7. Three numbers that differ by arithmetical proportion, the double of the mean, or middle number, is equal to the ſum of the extremes.

So 9, 12, and 15, being given, the double of the mean 12 (*viz.* 24) is equal to the ſum of the two extremes 9 and 15.

8. Four numbers that differ by arithmetical proportion, either continued or interrupted, the ſum of the two means is equal to the ſum of the two extremes.

So 9, 12, 18, 21, being given, the ſum of 12 and 18 will be equal to the ſum of 9 and 21, *viz.* 30. Alſo 6, 8, 14, 16, being given, the ſum of 8 and 14 is equal to the ſum of 6 and 16, *viz.* 22, *&c. Vid. Wing. arith. cap.* 35.

9. Geometrical proportion (by ſome called *geometrical progreſſion*) is, when divers numbers differ according to right reaſon.

So 1, 2, 4, 8, 16, 32, 64, *&c.* differ by double reaſon; and 3, 9, 27, 81, 243, 729, differ by triple reaſon; 4, 16, 64, 256, *&c.* differ by quadruple reaſon, *&c.*

10. In any numbers that increaſe by geometrical proportion, if you multiply the laſt term by the quotient of any one of the terms divided by another of the terms, which being leſs, is next unto it; and having deducted or ſubtracted the firſt term out of that product, divide the remainder by a number that is an unit leſs than the ſaid quotient, the laſt quote will give the ſum of all the terms.

So

So 1, 2, 4, 8, 16, 32, 64, being given; firſt, I take one of the terms, *viz.* 8, and divide it by the term, which is leſs, and next to it, *viz.* by 4, and the quotient is 2; by which I multiply the laſt term 64, and the product is 128; from whence I ſubtract the firſt term, *viz.* 1, the remainder is 127; which dived by the quotient 2, made leſs by 1, *viz.* 1, the quote is 127, for the ſum of all the given terms; as by the work in the margin.

```
     64
 4) 8 (2
   ----
    128
      1
   ----
 1) 127 (127
```

So if 4, 16, 64, 256, 1024, were given, the ſum of all the terms will be found to be 1364. For firſt, I divide 64, one of the terms, by the next leſſer term, and the quotient is 4; by which I multiply the laſt term, 1024, and it produceth 4096; from whence I ſubtract the firſt term, 4, and the remainder is 4092; which I divide by the quote, leſs by 1, *viz.* 3, and the quote is 1364, for the total ſum of all the terms; as *per* margin.

```
      1024
 16) 64 (4
     ----
     4096
        4
     ----
 3) 4092 (1364
```

So likewiſe if 2, 6, 18, 54, 162, 486, were given, the ſum or total of all the terms will be found to be 728. See the work.

```
      486
 6) 18 (3
    -----
     1458
        2
    -----
 2) 1456 (728
```

11. Three geometrical proportionals being given, the ſquare of the mean is equal to the rectangle or product of the extremes.

So 8, 16 32, being given, the ſquare of the mean, *viz.* 16, is 256, which is equal to the product of the extremes 8 and 32; for 8 times 32 is equal to 256.

12. Of four geometrical proportional numbers given, the product of the two means, is equal to the product of the two extremes.

So 8, 16, 32, 64, being given, I ſay, that the pro-

duct

duct of the two means, *viz.* 16 times 32, which is 512, is equal to 8 times 64, the product of the extremes.

Also if 3, 9, 21, 63, were given, which are interrupted, I say, 9 times 21 is equal to 3 times 63, which is equal to 189.

From hence ariseth that precious gem in arithmetic, which for the excellency thereof is called the *golden rule*, or *rule of three.*

C H A P. X.

The single rule of Three direct.

1. THE rule of three (not undeservedly called *the golden rule*) is that by which we find out a fourth number in proportion unto three given numbers, so as this fourth number sought may bear the same rate, reason, or proportion, to the third (given) number as the second doth to the first; from whence it is called *the rule of proportion.*

2. Four numbers are said to be proportional, when the first containeth, or is contained by the second, as often as the third containeth, or is contained by the fourth. *Vid. Wing. arith. cap.* 8. *sect.* 4.

So these numbers are said to be proportionals, *viz.* 3, 6, 9, 18; for as often as the first number is contained in the second, so often is the third contained in the fourth, *viz.* twice. Also 9, 3, 15, 5, are said to be proportionals: for as often as the first number containeth the second, so often the third number containeth the fourth, *viz.* 3 times.

3. The rule of three is either simple or compound.

4. The simple (or single) rule of three, consisteth of 4 numbers; that is to say, it hath three numbers given to find out a fourth. And this is either direct or inverse. *Vid. Alsted. math. lib.* 2. *cap.* 13.

5. The single rule of three direct, is, when the proportion of the first term is to the second, as the third is to the fourth; or when it is required that the number sought, *viz.* the fourth number, must have the same proportion to the second, as the third hath to the first.

6. In

6. In the rule of three, the greateſt difficulty is (after the queſtion is propounded) to diſcover the order of the three terms, *viz.* which is the firſt, which is the ſecond, and which the third. Which that you may underſtand, obſerve, That, of the three given numbers, two always are of one kind and the other is of the ſame kind with the proportional number that is ſought; as in this queſtion, *viz.* If 4 yards of cloth coſt 12 ſhillings, what will 6 yards coſt at that rate? Here the two numbers of one kind are 4 and 6, *viz.* they both ſignify ſo many yards; and 12 ſhillings is the ſame kind with the number ſought, for the price of 6 yards is ſought.

Again obſerve, that of the three given numbers, thoſe two that are of the ſame kind, one of them muſt be the firſt, and the other the third; and that which is of the ſame kind with the number ſought, muſt be the ſecond number in the rule of three. And that you may know which of the ſaid numbers to make your firſt, and which your third, know this, that to one of theſe two numbers there is always affixed a demand, and that number upon which the demand lieth, muſt always be reckoned the third number: As in the forementioned queſtion, the demand is affixed to the number 6; for it is demanded, what 6 yards will coſt; and therefore 6 muſt be the third number, and 4, which is of the ſame denomination or kind with it, muſt be the firſt: and conſequently the number 12 muſt be the ſecond. And then the numbers being placed in the forementioned order, will ſtand as followeth, *viz.*

Yards.		*s.*		*Yards.*
4	:	12	: :	6

7. In the rule of three direct, (having placed the number as is before directed), the next thing to be done will be, to find out the fourth number in proportion: which that you may do, multiply the ſecond number by the third, and divide the product thereof by the firſt; or (which is all one) multiply the third term (or number) by the ſecond, and divide the product thereof by the firſt; and the quotient thence ariſing is the fourth number in a direct proportion; and is the number ſought, or anſwer to the queſtion; and is of the ſame denomination

nation that the second number is of. As thus: Let the same question be again repeated, *viz.* If 4 yards of cloth cost 12 shillings, what will 6 yards cost?

Having placed the numbers according to the sixth rule of this chapter, I multiply (the second number) 12 by (the third number) 6, and the product is 72; which product I divide by (the first number) 4, and the quotient thence arising is 18: which is the fourth proportional, or number sought, *viz.* 18 shillings, (because the second number is shillings); which is the price of 6 yards, as was required by the question. See the work on the margin.

```
Yards.   s.   Yards.   s.
If 4  :  12  ::  6  :  18
          6
         --
     4)  72  (18 shillings
          4
         --
         32
         32
         --
         (0)
```

Quest. 2. If 7 C. of pepper cost 21 l. how much will 16 C. cost at that rate?

To resolve this question, I consider, that (according to the sixth rule of this chapter) the terms or numbers ought to be placed thus, *viz.* the demand lying upon 16 C. it must be the third number, and that of the same kind with it must be the first, *viz.* 7 C. and 21 l. (being of the same kind with the number sought) must be the second number in this question. Then I proceed according to the seventh rule, and multiply the second number by the third, *viz.* 21 by 16, and the product is 336; which I divide by the first number 7, and the quotient is 48 l. which is the value of 16 C. of pepper at the rate of 21 l. for 7 C. See the work on the margin.

```
C.     l.      C.
7  :   21  ::  16
       16
      ---
      126
      21
      ---
  7)  336  (48 l.
      28
      ---
       56
       56
      ---
      (0)
```

8. If when you have divided the product of the second and third numbers by the first, any thing remain

after division is ended, such remainder may be multiplied by the parts of the next inferior denomination that are equal to an unit (or integer) of the second number in the question; and the product thereof being divided by the first number in the question, the quotient is of the same denomination with the parts by which you multiplied the remainder, and is part of the fourth number which is sought. And furthermore, if any thing remain after this last division is ended, multiply it by the parts of the next inferior denomination, equal to an unit of the last quotient, and divide the product by the same divisor, *viz.* the first number in the question, and the quote is still of the same denomination with your multiplier. Follow this method, until you have reduced your remainder into the lowest denomination, *&c.* An example or two will make this rule very plain, which may be these following.

Quest. 3. If 13 yards of velvet (or any other thing) cost 21 l. what will 27 yards of the same cost at that rate?

Having ordered and wrought my numbers according to the sixth and seventh rules of this chapter, I find the quotient to be 43 l. and there is a remainder of 8; so that I conclude the price of 27 yards to be more than 43 l.: and to the intent that I may know how much more, I work according to the foregoing rule, *viz.* I multiply the said remainder 8 by 20, (because the second number in the question was pounds), and the product is 160; which divided by the first number, *viz.* 13, it quotes 12, which are 12 shillings; and there is yet a remainder of 4: which I multiply by 12, (because the last quotient was shillings), and the product is 48; which I divide by 13 (the first number), and the quotient is 3 d.; and yet there remaineth 9: which I multiply by 4, and the product is 36; which divided again by 13, it quotes 2 farthings; and there is yet a remainder of 10: which (because it cometh not to the value of a farthing) may be neglected: or rather set after the 2 farthings over the divisor, with a line between them; and then (by the 21st and 22d definitions of the first chapter of this book) it will be $\frac{10}{13}$ of a farthing. So that I conclude, that if 13 yards of velvet cost 21 l. 27 yards of the same will cost 43 l. 12 s. 3 d. $2\frac{10}{13}$ qrs.
which

which fraction is ten thirteenths of a farthing. See the operation as followeth.

	Yards,		*l.*		*Yards.*
If	13	:	21	::	27

27
———
147
42
———
13) 567 (43 l.
52
———
47
39
———
Remains 8
Multiply 20
———
13) 160 (12 s.
13
———
30
26
———
Remains 4
Multiply 12
———
13) 48 (3 d.
39
Remans 9
Multiply 4
13) 36 ($2\frac{10}{13}$ qrs.
26
—
Remains 10 *Facit* 43—12—3—$2\frac{10}{13}$ (*l. s. d. qrs.*)

Quest. 4. Another example may be this following, *viz.* If 14 lb. of tobacco cost 27 s. what will 478 lb. cost at that rate?

Work according to the last rule, and you will find it to amount to 921 s. 10 d. $1\frac{2}{14}$ qrs.; and, by the fifth rule of the 8th chapter, 921 s. may be reduced to 46 l. 1 s.: so that then the whole worth or value of the 478 lb. will be 46 l. 1 s. 10 d. $1\frac{2}{14}$ qrs. The whole work followeth.

	lb.	*s.*		*lb.*	
If	14	: 27	::	478	
				27	
				3346	
				956	2|0
			14)	12906	(92|1 (46 l.
				126	8
				30	12
				28	12
				26	1 s.
				14	
		Remains		12	
		Multiply		12	
				24	
				12	
			14)	144	(10 d.
				14	
		Remains		4	
		Multiply		4	
			14)	16	($1\frac{2}{14}$ qrs.
				14	
		Remains		2	

l. *s.* *d.* *qrs.*
Facit 46—1—10—$1\frac{2}{14}$

9. In

9. In the rule of three it many times happeneth, that although the first and third numbers be homogeneal, that is, of one kind, as both money, weight, measure, *&c.* yet they may not be of one denomination, or perhaps they may both consist of many denominations: in which case you are to reduce both numbers to one denomination; and likewise your second number (if it consisteth at any time of divers denominations) must be reduced to the least name mentioned, or lower if you please: which being done, multiply the second and third together, and divide by the first, as is directed in the seventh rule of this chapter.

And note, That always the answer to the question is in the same denomination that your second number is of, or is reduced to, as was hinted before.

Quest. 5. If 15 ounces of silver be worth 3 l. 15 s. what are 86 ounces worth at that rate?

In this question, the numbers being ordered according to the sixth rule of this chapter, the first and third numbers are ounces; and the second number is of divers denominations, *viz.* 3 l. 15 s. which must be reduced to shillings, and the shillings multiplied by the third number, and the product divided by the first, gives you the answer in shillings, *viz.* 430 shillings, which are reduced to 21 l. 10 s.

```
      oz.      l.     s.          oz.
   If 15   :   3 ——— 15    ::     86
               20
               ——
               75
               86
             ————
              450
             600
             ———— (2|0    l.
       15) 6450 (43|0  (21 — 10
             60      4
             ——      —
             45      3
             ——      2
             (0)     —
                     10 shillings
```

In resolving the last question, the work would have been the same, if you had reduced your second number into pence; for then the answer would have been 5160 pence, equal to 21 l. 10 s.; or if you had reduced the second number into farthings, the quotient or answer, would have been 20640 farthings, equal to the same; as you may prove at your leisure.

Quest. 6. If 8 lb. of pepper cost 4 s. 8 d. what will 7 C. 3 qrs. 14 lb. cost?

In this question the first number is 8 lb. and the third is 7 C. 3 qrs. 14 lb. which must be reduced to the same denomination with the first, *viz.* into pounds, and the second number must be reduced into pence; then multiply and divide according to the seventh rule foregoing, and you will find the answer to be 6174 pence, which is reduced into 25 l. 14 s. 6 d.

	lb.		*s. d.*		*C. qrs. lb.*
If	8	:	4—8	:	7—3—14
			12		4
			56		31
					28
					252
					63
					882
					56 second number
					5292
					4410

8) 49392 (6174   12) (6174 (514   2|0) (51|4 (25—14—6   *l. s. d.*

48   60   4

13   17   11

8   12   10

59   54   14 s.

56   48

32   6 d.

32

(0)   *Facit* 25—14—6 *l. s. d.*

Quest. 7. If 3 C. 1 qr. 14 lb. of raisins cost 9 l. 9 s. what will 6 C. 3 qrs. 20 lb. of the same cost?

Here the first and third numbers, each consist of divers denominations, but must be brought both into one denomination, *&c.* as you see in the operation that followeth. The answer is 388 s. which is reduced into 19 l. 8 s.

C. qr. lb.	*l. s.*	*C. qrs. lb.*		
If 3—1—14 :	9—9 : :	6—3—20		
4	20	4		
13	189	27		
28		28		
108		216		
27		56		
378 pounds		776 pounds		
		189 second number		
		6984		
		6208		
		776	2|0)	*l. s.*
	378)	146664	(38|8	(19—8.
		1134	2	
		3326	18	
		3024	18	
	l. s.	3024	8 s.	
	Facit. 19—8.	3024		
		(0)		

Quest. 8. If in 4 weeks I spend 13 s. 4 d. how long will 53 l. 6 s. last me at that rate?

Ans. 2238 days, equal to 6 years, 48 days. See the work in the following page.

If

	s.	*d.*		*w.*		*l.*	*s.*
If	13	—4	:	4	: :	53	—6

```
    13——4  :  4  : :  53——6
    12        7         20
    ——        —      ————
    30     28 days    1066
   13                   12
  ————              ——————
   160                2132
                     1066
                    ——————
                     12792 pence
                        28 second number
                    ——————
                    102336
                    25584
                    ——————        365)
            16|0)  35817|6 (2238 (6 years
                    ....         2190
                    32           ————
                   ———      Rem. 48 days
                    38
                    32
                   ———
                    61        Years.   days.
                    48    Facit 6———48 96/160
                   ———
                   137
                   128
                   ———
           Remains  96
```

Quest. 9. Suppose the yearly rent of a house, a yearly pension or wages be 73 l. I desire to know how much it is *per* day?

Here you are to bring the year into days, and say, If 365 days require 73 l. what will 1 day require?

Now, when you come to multiply 73 by one, the product is the same; for 1 neither multiplieth nor divideth; and 73 cannot be divided by 365, because the divisor is bigger than the dividend; wherefore bring the 73 l. into

shillings,

shillings, and they make 1460; which divide by the first number 365, and the quote is 4 shillings for the answer: as you see in the work.

Days.		*l.*		*Day.*
If 365	:	73	: :	1
		20		

365) 1460 (4 s.
1460

(0)

Facit 4 s. *per* day.

Quest. 10. A merchant bought 14 pieces of broad-cloth, each piece containing 28 yards, for which he gave after the rate of 13 s. $6\frac{1}{2}$ d. *per* yard. Now, I desire to know how much he gave for the 14 pieces at that rate?

First, find out how many yards are in the 14 pieces; which you will do, if you multiply the 14 pieces by 28, (the number of yards in a piece), and it makes 392. Then say, If 1 yard cost 13 s. $6\frac{1}{2}$ d. what will 392 yards cost? Work as followeth; and the answer you will find to be 127400 halfpence; which reduced, make 265 l. 8 s. 4 d.: for after you have multiplied your second and third numbers together, the product is 127400; which, according to the seventh rule, should be divided by the first number: but the first number is 1, which neither multiplieth nor divideth; and therefore the quotient or fourth number is the same with the product of the second and third; which is in halfpence, because the second number was so reduced. See the work as followeth.

```
                          28
                          14
                         ----
                         112
                         28
                         ----
                         392 yards in the 14 pieces

Yard.     s.   d.      Yards.
If 1  :  13—6½    ::   392
          12           325 the second number
         ----         -----
          32           1960
          13           784
         ----         1176
         162          ---------  2|0    l.
           2     24) 127400 (530|8 (265
         ----         ....     ...
Halfpence 325          120       4
                      -----
                        74      13
                        72      12
                      -----   -----
                       200      10
                       192      10
                      -----   -----
                                8 shillings
      l.   s.  d.    Rem. 8 halfpence, or 4 d.
Facit 265—8—4
```

Quest. 11. A draper bought 420 yards of broad-cloth, and gave for it after the rate of 14 s. 10¾ d. *per* ell English. Now, I demand how much he paid for the whole after that rate?

Bring your ells into quarters, and your given yards into quarters; the ell English is 5 quarters, and 420 yards are 1680 quarters: then say, If 5 quarters cost 14 s. 10¾ d. (or 715 farthings), what will 1680 quarters cost? *Facit* 250 l. 5 s. See the operation following.

Ell.

Ell.	*Yards.*
1	420
5	4
5	1680 qrs.

Qrs. *s.* *d.* *Qrs.*
If 5 : 14—$10\frac{1}{4}$: : 1680

```
       12          715
       --          ---
       28          840
       15         168
      ---        1176
      178 d.     ------ 96|0)
        4     5) 1201200 (24024|0 (250 l.
      ---        ......      ...
      715 qrs.    10        192
                  --        ---
                  20        482
                  20        480
                  --        ---
                   12  Rem. 240 qrs. or 5 s.
                   10
                   --
                   20
                   20
                   --
 l.   s.
Facit 250—5        (0)
```

Quæst. 12 A draper bought of a merchant 50 pieces of kersey, each piece containing 34 ells Flemish (the ell Flemish being 3 quarters of a yard), to pay after the rate of 8 s. 4 d. *per* ell English; I demand how much the 50 pieces cost him at that rate?

First find out how many ells Flemish are in the 50 pieces; by multiplying 50 by 34, the product is 1700; which bring into quarters by 3, it makes 5100 quarters; then proceed as in the last question, and the answer you will find to be 102000 pence, or 425 l. See the operation as followeth.

If

Qrs. *s.* *d.* *Qrs.*
If 5 : 8—4 : : 5100 50
12 100 34

100 d. 5) 510000 (102000 *d.* 20
15
5
1700 ells Fle.
10 3
10
5100
(0) 2|0 *l.*
12) 102000 (850|0 (425
96 8
60 5
60 4
(0) 10
10
Facit 425 l. (0)

Quest. 13. A goldsmith bought a wedge of gold which weighed 14 lb. 3 oz. 8 pw. for the sum of 514 l. 4 s. I demand what it stood him in *per* ounce? *Ans.* 60 shillings, or 3 l. See the work.

lb. *oz.* *pw.* *l.* *s.* *oz.*
If 14—3—8 : 514—4 : : 1
12 20 20

31 10284 shillings 20 pw.
14 20
2|0)
171 oz. 3428) 205680 (6|0 (3 l.
20 6
20568 *s.* *l.*
3428 pw. (0) *Facit.* 60 or 3
(c)

Quest.

Quest. 14. A grocer bought 4 hogsheads of sugar, each weighing neat 6 C. 2 qrs. 14 lb. which cost him 2 l. 8 s. 6 d. *per* C. I demand the value of the 4 hogsheads at that rate?

First find the weight of the 4 hogsheads; which you may do by reducing the weight of one of them into pounds, and multiply them by 4. (the number of hogsheads), and they make 2968 lb.: then say, If 1 C. or 112 lb. cost 2 l. 8 s. 6 d. what will 2968 lb. cost? *Facit* 64 l. 5 s. 3 d. as by the operation.

```
                                                    C.   qrs.  lb.
                                                    6——2——14
                                                     4
                                                    ———
                                                    26
                                                    28
                                                    ———
lb.      l. s. d.           lb.                     212
If 112 : 2—8—6   : :        2968                    53
         20                  582                    ———
         ——                 ————                    742 lb. in 1 hhd.
         48                 5936                      4 hhds.
         12                23744                    ———
         ———              14840                     2968 lb. in 4 hhds.
         102              ———————       12)          2|0)
         48       112)    1727376      (15423       (128|5   (64 l.
         ———              .....        .....         ...
         582              112           12           12
                          ———           ——           ——
                           607          34            8
                           560          24            8
                           ———          ——           ——
                            473         102           5 shillings
                            448          96
                            ———          ——
                             257         63
                             224         60
                             ———         ——
                              336         3 pence
                              336
                              ———                      l.  s. d.
                              (0)            Facit    64—5—3
```

Quest. 15. A draper bought of a merchant 8 packs of cloth, each containing 4 parcels, and each parcel 10 pieces, and each piece 26 yards, and gave after the rate of 4 l. 16 s. for 6 yards; now I desire to know how much he gave for the whole? *Ans.* 6656 l.

First find out how many yards there were in the 8 packs, and by the following work you will find there are 8320 yards: then say, If 6 yards cost 4 l. 16 s. what will 8320 yards cost? *&c.*

```
                                             8 packs
                                             4
                                            ---
                                            32 parcels
                                            10
                                            ---
                                           320 pieces
                                           26
                                           ----
Yards.    l.   s.        Yards.            192
If 6  :   4—16    ::     8320               64
          20              96               ----
          --             -----             8320 yards
          96             4992
                        7488
                        -----   2|0
                6)      798720 (13312|0 (6656 l.
                        ......   ....
                        6         12
                        -         --
                        19        13
                        18        12
                        --        --
                        18        11
                        18        10
                        --        --
                         7        12
                         6        12
                        --        --
                        12        (0)
                        12
                        --
                        (0)
```

Facit 6656 l.

10. By

16. By this time the learner is, as I suppose, well exercised in the practic and theoretic of the rule of three direct; but at his leisure he may look over the following questions, whose answers are given, but the operation purposely omitted, as a touchstone for the learner, thereby to try his ability in what hath been delivered in the former rules.

Quest. 16. If 24 lb. of raisins cost 6 s. 6 d. what will 18 frails cost, each weighing neat 3 qrs. 18 lb.? *Ans.* 24 l. 17 s. 3 d.

Quest. 17. If an ounce of silver be worth 5 shillings, what is the price of 14 ingots, each ingot weighing 7 lb. 5 oz. 10 pw.? *Ans.* 313 l. 5 s.

Quest. 18. If a piece of cloth cost 10 l. 16 s. 8 d. I demand how many ells English there are in the same, when the ell at that rate is worth 8 s. 4 d.? *Ans.* 26 ells English.

Quest. 19. A factor bought 84 pieces of stuffs, which cost him in all 537 l. 12 s. at 5 s. 4 d. *per* yard, I demand how many yards there were in all, and how many ells English were contained in a piece of the same? *Ans.* 2016 yards in all, and $19\frac{1}{5}$ ells English *per* piece.

Quest. 20. A draper bought 242 yards of broad-cloth, which cost him in all 254 l. 10 s. for 86 yards, of which he gave after the rate of 21 s. 4 d. *per* yard, I demand how much he gave *per* yard for the remainder? *Ans.* 20 s. $10\frac{64}{156}$ d. *per* yard.

Quest. 21. A factor bought a certain quantity of serge and shalloon, which, together, cost him 226 l. 14 s. 10 d. the quantity of serge he bought was 48 yards at 3 s. 4 d. *per* yard; and for every two yards of serge he had 5 yards of shalloon; I demand how many yards of shalloon he had, and how much the shalloon cost him *per* yard? *Ans.* 120 yards of shalloon at 1 l. 16 s. $5\frac{58}{120}$ d. *per* yard.

Quest. 22. An oilman bought 3 tuns of oil, which cost him 151 l. 14 s.; and so it chanced that it leaked out 85 gallons; but he is minded to sell it again, so as that he may be no loser by it, I demand how he must sell it *per* gallon? *Ans.* At 4 s. $6\frac{174}{671}$ d. *per* gallon.

Quest. 23. A merchant bought 6 packs of cloth, each pack containing 12 cloths, which at 8 s. 4 d. *per* ell Flemish,

mish, cost 1080 l. I demand how many yards there were in each cloth? *Ans.* 27 yards in each cloth.

Quest 24. A gentleman hath 536 l. *per annum,* and his expenses are, one day with another, 18 s. 10 d. 3 qrs. I desire to know how much he layeth up at the year's end? *Ans.* 191 l. 3 s. 1 qr.

Quest. 25. A gentleman expendeth daily, one day with another, 27 s. $10\frac{1}{2}$ d. and at the year's end layeth up 340 l. I demand how much is his yearly income? *Ans.* 848 l. 14 s. $4\frac{1}{2}$ d.

Quest. 26. If I sell 14 yards for 10 l. 10 s. how many ells Flemish shall I sell for 283 l. 17 s. 6 d. at that rate? *Ans.* $504\frac{2}{3}$ ells Flemish.

Quest. 27. If 100 l. in 12 months gain 6 l. interest, how much will 75 l. gain in the same time, and at the same rate? *Ans.* 4 l. 10 s.

Quest. 28. If 100 l. in 12 months gain 6 l. interest, how much will it gain in 7 months at that rate? *Ans.* 3 l. 10 s.

Quest. 29. A certain usurer put out 75 l. for 12 months, and received principal and interest 81 l. I demand at what rate *per cent.* he received interest? *Ans.* At 8 *per cent.*

Quest. 30. A grocer bought two chests of sugar, the one weighed neat 17 C. 3 qrs. 14 lb. at 2 l. 6 s. 8 d. *per* C. the other weighed neat 18 C. 1 qr. 21 lb. at $4\frac{1}{2}$ d. *per* lb. which he mingled together; now I desire to know how much a C. weight of this mixture is worth? *Ans.* 2 l. 4 s. 3 d. $2\frac{1078}{4067}$ qrs.

Quest. 31. Two men, *viz.* A and B, departed both from one place; the one goes east, and the other west; the one travelleth 4 miles a-day, and the other 5 miles a-day; how far are they distant the 9th day after their departure? *Ans.* 81 miles.

Quest. 32. A flying every day 40 miles, is pursued the 4th day after by B, posting 50 miles a-day: now the question is, In how many days, and after how many miles travel will A be overtaken? *More's aritemetic, cap.* 8. *quest.* 7.

Ans. B overtakes him in 12 days, when they have travelled 600 miles.

11. The general and first effect of the rule of three direct

direct is contained in the definition of the same; that is, to find a fourth number in proportion to three numbers given, as hath been fully shewn in all the foregoing examples.

The second effect is, by the price or value of one thing, to find the price or value of many things of like kind.

The third effect is, by the price or value of many things, to find the price of one; or by the price of many things, (the said price being one), to find the price of many things of like kind.

The fourth effect is, by the price or value of many things, to find the price or value of many things of like kind.

The fifth effect is, thereby to reduce any number of monies, weights, or measures, the one sort into the other, as in the rules of reduction contained in the eighth chapter foregoing. Examples of its various effects have been already given.

The proof of the rule of three direct.

12. The rule of three direct is thus proved, *viz.*

Multiply the 1st number by the 4th, and note the product; then multiply the 2d number by the 3d: and if this product is equal to the product of the 1st and 4th, then the work is rightly performed; otherwise it is erroneous.

So the first question of this chapter (whose answer or 4th number we found to be 18s.) is thus proved, *viz.* The 1st number is 4; which multiplied by 18 (the 4th) produceth 72; and the 2d and 3d numbers are 12 and 6, which multiplied together, produce 72, equal to the product of the 1st and 4th; and therefore I conclude the work to be rightly performed.

Always observing, that if any thing remain after you have divided the product of the 2d and 3d numbers by the 1st, such remainder, in proving the same, must be added to the product of the 1st and 4th numbers, whose sum will be equal to the product of the 2d and 3d; the 2d number being of the same denomination with the 4th, and the 1st of the same denomination with the 3d.

So the fourth question of this chapter being again repeated, *viz.* If 14 lb. of tobacco cost 27 s. what will 478 lb. cost at that rate? the answer (or 4th number) was 46 l. 1 s. 10 d. 1 qr. $\frac{2}{14}$: which is thus proved, *viz.* Bring the fourth number into farthings, and it makes 44249; which multiplied by the 1st number 14, produceth 619488, (the remainder 2 being added thereto.) Then, because I reduce my 4th number into farthings, I reduce my 2d (*viz.* 27 s.) into farthings, and they are 1296; which multiplied by the 3d number 478, their product is 619488, equal to the product of the 1st and 4th numbers; wherefore I conclude the operation to be true. This is an infallible way to prove the rule of three direct; and it is deduced from the 12th section of the 9th chapter of this book.

Thus much concerning the single rule of three direct; and I question not, but that by this time the learner is sufficiently qualified to resolve any question pertinent to this rule, not depending upon fractions or geometrical magnitudes. Those that are desirous to see the demonstration of this rule, let them read the sixth chapter of the ingenious Mr Kersey's appendix to Mr Wingate's arithmetic; or the sixth chapter of Mr Oughtred's incomparable *clavis mathematica*: by both which authors this rule is largely demonstrated, being grounded upon the 16th proposition of the sixth book of Euclid's elements.

CHAP. XI.

The single rule of Three inverse.

1. THE golden rule, or rule of three inverse, is, when there are three numbers given to find a fourth in proportion to the three given numbers, so as the fourth proceeds from the second according to the same rate, reason, or proportion, that the first proceeds from the third. Or the proportion is,

As the third number is in proportion to the first, so is the second to the fourth. *Alsted. math. lib.* 2. *cap.* 14.

So if the three numbers given were 8, 12, and 16, and

and it were required to find a fourth number in an inverted proportion to theſe; I ſay, that as 16 (the third number) is the double of the firſt term or number 8; ſo muſt 12 (the ſecond number) be the double of the fourth; ſo will you find the fourth term or number to be 6. And as, in the rule of three direct, you multiply the ſecond and third together, and divide their product for a fourth proportional number: ſo,

2. In the rule of three inverſe, you muſt multiply the ſecond term by the firſt (or firſt term by the ſecond), and divide the product thereof by the third term, ſo the quotient will give you the fourth term ſought in an inverted proportion. The ſame order being obſerved in this rule, as in the rule of three direct, for placing and diſpoſing of the given numbers, after your numbers are placed, in order to know whether your queſtion is to be reſolved by the rule direct or inverſe, obſerve the general rule following.

3. When your queſtion is ſtated, and your numbers orderly diſpoſed, conſider, in the firſt place, whether the fourth term, or number ſought, ought to be more or leſs than the ſecond term; which you may eaſily do: and if it is required to be more or greater than the ſecond term, then the leſſer extreme muſt be your diviſor; but if it require leſs, then the higheſt extreme muſt be your diviſor; (in this caſe the firſt and third numbers are called *extremes* in reſpect of the ſecond); and having found out your diviſor, you may know whether your queſtion belong to the rule direct or inverſe: for if the third term be your diviſor, then it is inverſe; but if the firſt term be your diviſor, then it is direct: As in the following queſtions.

Queſt. 1. If 8 labourers can do a certain piece of work in 12 days, in how many days will 16 labourers do the ſame? *Anſ.* In 6 days.

Having placed the numbers according to the ſixth rule of the tenth chapter, I conſider that if 8 men can finiſh the work in 12 days, 16 men will do it in leſs or fewer days than 12; therefore the biggeſt ex-

lab.		*days.*		*lab:*
8	:	12	: :	16
		8		
16)		96 (6 days		
		96		
		(0) *Facit* 6 days.		

treme

treme must be the divisor, which is 16; and therefore it is the rule of three inverse: wherefore I multiply the first and second numbers together, *viz.* 8 by 12, and their product is 96; which divided by 16, quotes 6 days for the answer. And in so many days will 16 labourers perform a piece of work, when 8 man can do it in 12 days.

Quest. 2 If, when the measure (*viz.* a peck) of wheat cost 2 shillings, the penny-loaf weighed (according to the standard statute, or law of England) 8 ounces, I demand how much it will weigh when the peck is worth 1 s. 6 d. according to the same rate or proportion? *Answ.* 10 oz. 13 pw. 8 gr.

Having placed and reduced the given numbers according to the sixth and ninth rules of the tenth chapter, I consider, that 1 s. 6 d. *per* peck, the penny loaf will weigh more than at 2 s. *per* peck; for as the price decreaseth, the weight increaseth; and as the price increaseth, so the weight diminishes: wherefore because the third term requires more than the first, the lesser extreme must be the divisor, *viz.* 1s. 6d. or 18d.: and having finished the work, I find the answer to be 10 oz. 13 pw. 8 gr.; and so much will the penny-loaf weigh when the peck of wheat is worth 1 s. 6d. according to the given rate of 8 ounces, when the peck is worth 2 shillings. The work is plain in the margin.

```
 s.      oz.      s. d.
 2   :    8  : :  1—6
12       24      12
——       ——      ——
24       32      18
         16
         ——   oz. pw. gr.
    18) 192 (10—13—8
        18
        ———
   Rem. 12
        20
        ———
    18) 240 (13 pw.
        18
        ——
        60
        54
        ——
   Rem.  6
        24
        ———
    18) 144 (8 gr.
        144
        ———
        (0)
```

Quest.

Quest. 3. How many pieces of money or merchandise, at 20s. *per* piece, are to be given or received for 240 pieces, the value or price of every piece being 12 shillings? *Ans.* 144 pieces. For if 12 s. require 240 pieces, then 20s. will require less; therefore the bigger extreme must be the divisor, which is the third number. See the work.

```
     s.    pieces.      s.
If 12  :    240   : :   20
             12
            ---
             48
            24
           -----
     2|0) 288|0 (144 pieces at 20 s. per piece
          . . .
          2
          --
           8
           8
           --
            8
            8
           --
           (0)
```

Quest. 4. How many yards of 3 quarters broad, are required to double or be equal in measure to 30 yards, that are 5 quarters broad? *Ans.* 50 yards. For say, If 5 quarters wide require 30 yards long, what length will 3 quarters broad require? Here I consider that 3 quarters broad will require more yards than 30; for the narrower the cloth is, the more in length will go to make equal measure with a broader piece.

```
qrs.   long.   qrs.
 5  :   30  : :  3
         5
       -----
   3)  150 (50 yards
        .
       15
       --
       (0)
```

Quest. 5. At the request of a friend, I lent him 200l. for 12 months; he promising to do me the like courtesy at my necessity; but when I came to request it of him, he could

could let me have but 150 l.: now I desire to know how long I may keep this money to make plenary satisfaction for my former kindness to my friend? *Ans.* 16 months. I say, If 200 l. will require 12 months, what will 150 l. require? 150 l. will require more time than 12 months; therefore the lesser extreme (*viz.* 150) must be the divisor; multiply and divide, and you will find the fourth inverted proportional to be 16. And so many months I ought to keep the 150 l. for satisfaction.

Quest. 6. If for 24 s. I have 1200 lb. weight carried 36 miles, how many miles shall 1800 lb. be carried for the same money? *Ans.* 24 miles.

Quest. 7. If for 24 s. I have 1200 lb. weight carried 36 miles, how many pound weight shall I have carried 24 miles for the same money? *Ans.* 1800 lb. weight.

Quest. 8. If 100 workmen in 12 days finish a piece of work or service, how many workmen are sufficient to do the same in 3 days? *Ans.* 400 workmen.

Quest. 9. A colonel is besieged in a town in which are 1000 soldiers, with provision of victuals only for 3 months; the question is, How many of his soldiers must he dismiss that his victuals may last the remaining soldiers 6 months? *Ans.* 500 he must keep, and dismiss as many.

Quest. 10. If 20 l. worth of wine is sufficient for the ordinary of 100 men, when the tun is sold for 30 l. how many men will the same 20 l. worth suffice when the tun is worth 24 l.? *Ans.* 125 men.

Quest. 11. How much plush is sufficient to line a cloak, which hath in it 4 yards of 7 quarters wide, when the plush is but 3 quarters wide? *Ans.* $9\frac{1}{3}$ yards of plush.

Quest. 12. How many yards of canvas that is ell wide, will be sufficient to line 20 yards of say that is 3 quarters wide? *Ans.* 12 yards.

Quest. 13. How many yards of matting that is two foot wide, will cover a floor that is 24 foot long and 20 foot wide? *Ans.* 240 foot.

Quest. 14. A regiment of soldiers consisting of 1000, are to have new coats, and each coat to contain 2 yards 2 quarters of cloth, that is 5 quarters wide, and they are to be lined with shalloon that is 3 quarters wide;

I

I demand how many yards of ſhalloon will line them? *Anſ.* 16666$\frac{2}{3}$ quarters or 4166$\frac{2}{3}$ yards.

Queſt. 15. A meſſenger makes a journey in 24 days when the day is 12 hours long, I deſire to know in how many days he will go the ſame when the day is 16 hours long? *Anſ.* In 18 days.

Queſt. 16. I borrowed of my friend 64 l. for 8 months, and he hath occaſion another time to borrow of me for 12 months, I deſire to know how much I muſt lend to make good his former kindneſs to me? *Anſ.* 42 l. 13 s. 4 d.

4. The general and firſt effect of the rule of three inverſe, is contained in the definition of the ſame; that is, to find a fourth term in a proportion inverted to the three numbers given.

The ſecond effect is, by two prices or values of two ſeveral pieces of money or merchandiſes known, to find how many pieces of the one price are to be given for ſo many of the other; and conſequently to reduce and exchange one ſort of money or merchandiſe into another: or contrariwiſe, to find the price of any piece given to be exchanged in an inverted proportion.

The third effect is, by two different prices of a meaſure of wheat bought or ſold, and the weight of the loaf of bread made anſwerable to one of the prices of the meaſure given, to find out the weight of the ſame loaf anſwerable to the other price of the ſaid meaſure given: or contrariwiſe, by the two ſeveral weights of the ſame priced loaf, and the price of the meaſure of wheat anſwerable to one of thoſe weights given, to find out the other price of the meaſure anſwerable to the other weight of the ſame loaf.

The fourth effect is, by two lengths and one breadth of two rectangular planes known, to find out another breadth unknown; or by two breadths and one length given, to find out another length unknown, in an inverted proportion.

The fifth effect is, by two times, and a capital ſum of money borrowed or lent, to find out another capital ſum anſwerable to one of the given times; or otherwiſe, by two capital ſums, and a time anſwerable to one of them

them given, to find out a time anſwerable to the other capital ſum in reciprocal reaſon.

The ſixth effect is, by two different weights of carriage, and the diſtance of the place in miles or in leagues given, to find another diſtance in miles anſwerable to the ſame price of payment; or otherwiſe by two diſtances in miles, and the weight anſwerable to one of the diſtances (being carried for a certain price), to find out the weight anſwerable to the other diſtance for the ſame price.

The ſeventh effect is, by two numbers of workmen, and the time anſwerable to one of the numbers of workmen given, to find out the time anſwerable to the other number of workmen in the performance of any work or ſervice: or contrariwiſe, by two times, and the workmen anſwerable to one of thoſe times given, to find out the number of workmen anſwerable to the other time, in the performance of any work or ſervice.

Alſo by two prices of proviſion, and the number of men or other creatures nouriſhed for a certain time anſwerable to one of the prices of proviſions given, to find out another number of men or other creatures anſwerable to the other price of the proviſion for the ſame time: or contrariwiſe, by two numbers of men or other creatures nouriſhed, and one price of proviſion anſwerable to one of the numbers of creatures given, to find out the other price of the ſame proviſion anſwerable to the other number of creatures, both being ſuppoſed to be nouriſhed for the ſame time, *&c.* as in the foregoing examples is fully declared.

To prove the operation of the rule of three inverſe, multiply the third and fourth terms together, and note their product; and multiply the firſt and ſecond together: and if their product is equal to the product of the third and fourth, then is the work truly wrought; but if it falleth out otherwiſe, then it is erroneous:

As in the firſt queſtion of this chapter, 16 (the third number) being multiplied by 6 (the fourth number), the product is 96; and the product of 8 (the firſt number) multiplied by 12 (the ſecond number) is 96, equal to the firſt product; which proves the work to be right.

And note, that if in diviſion any thing remain, ſuch remainder

remainder must be added to the product of the third and fourth terms; and if the sum be equal to the product of the first and second (the homogeneal terms being of one denomination), the work is right.

CHAP. XII.

The double Rule of Three direct.

WE have already delivered the rule of single proportion, and we come now to lay down the rules of plural proportion.

1. Plural proportion is, when more operations in the rule of three than one are required before a solution can be given to the question propounded: therefore in questions that require plurality in proportion, there are always given more than three numbers.

2. When there are given five numbers, and a sixth is required in proportion thereunto, then this sixth proportional is said to be found out by the double rule of three, as in the question following, *viz.*

If 100 l. in 12 months gain 6 l. interest, how much will 75 l. gain in 9 months?

3. Questions in the double rule of three, may be resolved either by two single rules of three, or by one single rule of three compounded of the five given numbers.

4. The double rule of three, is either direct, or else inverse.

5. The double rule of three direct is, when unto five given numbers a sixth proportional may be found out by two single rules of three direct.

6. The five given numbers in the double rule of three direct consist of two parts, *viz.* first, a supposition; and, secondly, a demand: the supposition is contained in the three first of the five given numbers, and the demand lies in the two last: As in the example of the second rule of this chapter, *viz.* If 100 l. in 12 months gain 6 l. interest, what will 75 l. gain in 9 months? Here the supposition is expressed in 100, 12, and 6; for it is said, If 100 l. in 12 months gain 6 l. interest: and the demand lieth in 75

 and

and 9; for it is demanded, how much 75 l. will gain in 9 months?

7. The next thing is to diſpoſe of the given numbers in due order and place, as a preparative for reſolution. Which that you may do, firſt, obſerve which of the given numbers in the ſuppoſition is of the ſame denomination with the number required; for that muſt be the ſecond number (in the firſt operation) of the ſingle rule of three; and one of the other numbers in the ſuppoſition (it matters not which) muſt be the firſt number; and that number in the demand which is of the ſame denomination with the firſt, muſt be the third number: which three numbers being thus placed, will make one perfect queſtion in the ſingle rule of three: As in the forementioned example, firſt, I conſider, that the number required in the queſtion, is the intereſt or gain of 75 l.; therefore that number in the ſuppoſition which hath the ſame name, *viz.* 6 l. which is the intereſt or gain of 100 l. muſt be the ſecond number in the firſt operation; and either 100 or 12 (it matters not which) muſt be the firſt number: but I will take 100; and then for the third number I put that number in the demand which hath the ſame denomination with 100, which is 75, (for they both ſignify pounds principal); and then the numbers will ſtand as you ſee in the margin.

100 : 6 : : 75

But if I had for the firſt number put the other number in the ſuppoſition, *viz.* 12, which ſignifies 12 months; then the third number muſt have been 9, which is the number in the demand which hath the ſame denomination with the firſt, *viz.* 9 months; and then they will ſtand as in the margin.

12 : 6 : : 9

There yet remain two numbers to be diſpoſed of; and thoſe are, one in the ſuppoſition, and another in the demand: that which is of the ſuppoſition, I place under the firſt of the three numbers; and the other, which is of the demand, I place under the third number; and then two of the terms in the ſuppoſition will ſtand one over the other in the

100	: 6 : :	75
12		9

Or thus,

12	: 6 : :	9
100		75

the first place, and the two terms in the demand will stand one over the other in the third place, as in the margin.

8. Having disposed or ordered the given numbers according to the last rule, we may proceed to a resolution. And, first, I work with the three uppermost numbers, which according to the first disposition are 100, 6, and 75; which is as much as to say, If 100 l. require 6 l. interest, how much will 75 l. require? which, by the third rule of the 11th chapter, I find to be direct, and by the seventh and eighth rules of the 10th chapter I find the fourth proportional number to be 4 l. 10s.; so that by the foregoing single question I have discovered how much interest 75 l. will gain in 12 months; the operation whereof followeth, on the left hand under the letter A. And having discovered how much 75 l. will gain in 12 months, we may by another question easily discover how much it will gain in 9 months; for this fourth number, thus found, I put in the middle between the two lowest numbers of the five, after they are placed according to the seventh rule of this chapter; and then it will be the second term in another question of the rule of three, the numbers being

m. *l.* *s.* *m.*
12 : 4—10 :: 9, where the first and third numbers are of one denomination, *viz.* both months, and may be thus expressed: If 12 months require 4 l. 10 s. interest, what will 9 months require? and by the third rule of the 11th chapter, I find it to be the direct rule; and by working according to the directions laid down in the seventh, eighth, and ninth rules of the 10th chapter, I find the fourth proportional number to the last single question, to be 3 l. 7 s. 6 d. which is the sixth proportional number to the five given numbers, and is the answer to the general question. The work of the last single question is expressed on the right side of the page, under the letter B, as followeth.

l. l. l.
100 : 6 :: 75
A. m. 12 9 m. B.

l. l. l.
If 100 : 6 :: 75
75
30
42
l. s.
1|00 4|50 (4—10
4
Rem. 50
Mult. 20
1|00) 10|00 (10s.

l. s.
Facit 4——10

Then say,
m. l. s. m.
If 12 : 4—10 :: 9
20
90 shillings
12
18
9
1080 pence
9
12) 2|0 l. s. d.
12) 9720 (810 (6|7 (3—7—6
96 72 6
12 90 7 s.
12 84
(0) 6 pence

l. s. d.
Facit 3——7——6

So that, by the foregoing operation, I conclude, that if 100 l. in 12 months gain 6 l. interest, 75 l. will gain 3 l. 7 s. 6 d. in 9 months after the same rate. The answer would have been the same if the five given numbers had been ordered according to the second method, *viz.* as you see in the margin.

12 : 6 :: 9
100 75

For, first, I say, If 12 months gain 6 l. what will 9 months gain? This question I find to be direct by the third rule of the 11th chapter; and by the seventh and eighth rules of the 10th chapter, I find the fourth proportional number to these three to be 4 l. 10 s.

Thus

Thus I have found out what is the intereſt of 100 l. for 9 months; and I am now to find the intereſt of 75 l. for 9 months: to effect which, I make this fourth number (found as before) to be my ſecond number in the next queſtion, and ſay, If 100 l. require 4 l. 10 s. what will 75 l. require? This queſtion I find, by the ſaid third rule of the 11th chapter, to be direct; and by the ſaid ſeventh, eighth, and ninth rules of the 10th chapter, I find the anſwer to be as before, *viz.* 3 l. 7 s. 6 d.

This rule hath been ſufficiently explained by the foregoing example; ſo that the learner may be able to reſolve the following, or any other queſtions pertinent to the double rule of three direct, whoſe anſwers are here given; but the operations are purpoſely omitted, to try the learner's ability in the knowledge of what has been before delivered.

Queſt. 2. A ſecond example in this rule may be as followeth, *viz.* A carrier receiving 42 ſhillings for the carriage of 3 C. weight 150 miles, I demand how much he ought to receive for the carriage of 7 C. 3 qrs. 14 l. 50 miles at that rate? *Anſ.* 36 s. 9 d.

Queſt. 3. If a regiment of 936 ſoldiers eat up 351 quarters of wheat in 168 days, I demand how many quarters of wheat 11232 ſoldiers will eat in 56 days at that rate? *Anſ.* 1404 quarters.

Queſt. 4. If 40 acres of graſs be mowed by 8 men in 7 days, how many acres ſhall be mowed by 24 men in 28 days? *Anſ.* 480 acres.

Queſt. 5. If 48 buſhels of corn, or other ſeed, yield 576 buſhels in 1 year, how much will 240 buſhels yield in 6 years at that rate; that is to ſay, if there were ſowed 240 buſhels every one of the 6 years? *Anſ.* 17280 buſhels.

Queſt. 6. If 40 ſhillings is the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days? *Anſ.* 768 ſhillings, or 38 l. 8 s.

Queſt. 7. If 14 horſes eat 56 buſhels of provender in 16 days, how many buſhels will 20 horſes eat in 24 days? *Anſ.* 120 buſhels.

Queſt. 8. If 8 cannons in one day ſpend 48 barrels of powder, I demand how many barrels 24 cannons will ſpend in 12 days at that rate? *Anſ.* 1728 barrels.

Queſt. 9. If in a family conſiſting of 7 perſons, there

are drunk out 2 kilderkins of beer in 12 days, how many kilderkins will there be drunk out in 8 days by another family consisting of 14 persons? *Ans.* 48 gallons, or 2 kilderkins and 12 gallons.

Quest. 10. An usurer put 75 l. out to receive interest for the same, and when it had continued 9 months, he received for principal and interest 78 l. 7 s. 6 d. I demand at what rate *per cent. per annum* he received interest? *Ans.* at 6 *per cent. per annum.*

CHAP. XIII.

The double Rule of Three inverse.

1. THE double rule of three inverse is, when a question in the double rule of three is resolved by two single rules of three, and one of those single rules falls out to be inverse, or requires a fourth number in an inverted proportion; for both questions are never inverse.

2. In all questions of the double rule of three, as well inverse as direct, you are, in the disposing of the five given numbers, to observe the seventh rule of the 12th chapter; and in resolving of it by two single rules, observe to make choice of your numbers for the first and second single questions according to the directions given in the eighth rule of the same chapter, as in the example following, *viz.*

Quest. 1. If 100 l. principal in 12 months gain 6 l. interest, what principal will gain 3 l. 7 s. 6 d. in 9 months?

This question is an inversion of the first question of the 12th chapter, and may serve for a proof thereof.

In order to a resolution, I dispose of the five given numbers according to the seventh rule of the last chapter; and being so disposed, they will stand as *per* margin.

m.		*l.*		*m.*	
12	:	100	::	9	
l.				*l. s. d.*	
6				3—7—6	

Or thus,

	l.		*l.*		*l. s. d.*	
	6	:	100	::	3—7—6	
m.	12				9	*m.*

Here

Here observe, that, according to the eighth rule of the 12th chapter, the first question, if you take it from the five numbers, as they are ordered or placed first, will be, If 12 months require 100 l. principal, what will 9 months require to make the same interest? This, according to the third rule of the 11th chapter, is inverse; and the answer will be found, by the second rule of the 11th chapter, to be 133 l. 6 s. 8 d. The second question then will be, If 6 l. interest require 133 l. 6 s. 8 d. principal, how much principal will 3 l. 7 s. 6 d. require? This is a direct rule; and the answer, in a direct proportion, is 75 l. See the work.

First I say,

m. *l.* *m.*
If 12 : 100 : : 9
12
———— *l.* *s.* *d.*
9) 1200 (133—6—8
...
9 *l.* *s.* *d.*
— *Facit* 133—6—8
30
27
——
30
27
——
(3)
20
—
9) 60 (6 s.
54
——
(6)
12
—
9) 72 (8 d.
72
—
(0)

Then

Then I say,

```
  l.        l.     s.    d.           l.    s.    d.
If 6   :   133 —— 6 —— 8   : :     3 —— 7 —— 6
 240        20                     20
 ———       ———                     ——
1440 d.    2666                    67
             12                    12
           ————                   ————
           5340                   140
          2666                    67
          ————                   ————
          32000                  810 d.
          810
          —————
           32
        256
        ——————          24|0)
144|0) 2592000|0   (1800|0 d. or 75 l.
        .....            ..
        144            168
        ———            ———
        1152           120
        1152           120
        ————           ———
         (0)           (0)
```

So that by the foregoing work I find, that if 6 l. interest be gained by 100 l. in 12 months, 3 l. 7 s. 6 d. will be gained by 75 l. in 9 months.

But if the resolution had been found out by the numbers as they are ranked in the second place, then the second question in the single rule would have been inverse, and the first question direct, and the conclusion the same with the first method, *viz.* 75 l.

Quest. 2. If a regiment, consisting of 936 soldiers, can eat up 351 quarters of wheat in 168 days, how many soldiers will eat up 1404 quarters in 56 days at that rate? *Ans.* 11232 soldiers.

Quest. 3. If 12 students in 8 weeks spend 48 l. I demand how many students will spend 288 l. in 18 weeks? *Ans.* 32 students.

Quest.

Quest. 4. If 48 l. serve 12 students 8 weeks, how many weeks will 288 l. serve 4 students? *Ans.* 144 weeks.

Quest. 5. If when a bushel of wheat cost 3 s. 4 d. the penny-loaf weigheth 12 ounces, I demand the weight of the loaf worth 9 d. when the bushel cost 10 s.? *Ans.* 36 ounces.

Quest. 6. If 48 pioneers in 12 days cast a trench 24 yards long, how many pioneers will cast a trench 168 yards long in 16 days? *Ans.* 252 pioneers.

Quest. 7. If 12 C. weight, being carrried 100 miles, cost 5 l. 12 s. I desire to know how many C. weight may be carried 150 miles for 12 l. 12 s. at that rate? *Ans.* 18 C.

Quest. 8. If when wine is worth 30 l. *per* tun, 20 l. worth is sufficient for the ordinary of 100 men, how many men will 4 l. worth suffice when it is worth 24 l. *per* tun? *Ans.* 25 men.

Quest. 9. If 6 men in 24 days mow 72 acres, in how many days will 8 men mow 24 acres? *Ans.* in 6 days.

Quest. 10. If when the tun of wine is worth 30 l. 100 men will be satisfied with 20 l. worth, I desire to know what the tun is worth, when 4 l. worth will satisfy 25 men at the same rate? *Ans.* 24 l. *per* tun.

C H A P. XIV.

The Rule of Three composed of five Numbers.

1. THE rule of three composed, is, when questions, wherein there are five numbers given to find a sixth in proportion thereunto, are resolved by one single rule of three composed of the five given numbers.

2. When questions may be performed by the double rule of three direct, and it is required to resolve them by the rule of three composed; first order or rank your numbers according to the seventh rule of the 12th chapter; then

The rule is,

Multiply the terms or numbers that stand one over the other

other in the first place, the one by the other, and make their product the first term in the rule of three direct. Then multiply the terms that stand one over the other in the third place, and place their product for the third term in the rule of three direct, and put the middle term of the three uppermost for a second term. Then having found a fourth proportional direct to these three, this fourth proportional so found shall be the answer required.

So the first question of the 12th chapter being proposed, *viz.* If 100 l. in 12 months gain 6 l. interest, what will 75 l. gain in 9 months? the numbers being ranked or placed as is there directed and done.

I multiply the two first terms, 100 and 12, the one by the other, and their product is 1200 for the first term. Then I multiply the two last terms 75 and 9 together, and their product is 675 for the third term. Then I say, As 1200 is to 6, so is 675 to the answer; which by the rule of three direct, will be found to be 3 l. 7 s. 6 d. as was before found.

3. But if the question be to be answered by the double rule of three inverse; then (having placed the five given terms as before) multiply the lowermost term of the first place, by the uppermost term of the third place, and put the product for the first term. Then multiply the uppermost term of the first place, by the lowermost term of the third place, and put the product for the third term; and put the second term of the three highest numbers for the middle term to those two. Then if the inverse proportion is found in the uppermost three numbers, the fourth proportional direct to these three shall be the answer. So the first question in the 13th chapter being stated, *viz.* If 100 l. principal in 12 months gain 6 l. interest, what principal will gain 3 l. 7 s. 6 d. in 9 months? state the numbers as is there directed in the first order, as in the margin.

m.		*l.*		*m.*
12	:	100	: :	9
l.				*l.* *s.* *d.*
6				3—7—6

Then reduce the 6 l. and 3 l. 7 s. 6 d. into pence; the 6 l. is 1440 d. and 3 l. 7 s. 6 d. is 810 d. Then multiply 1440 by 9, the product is 12960 for the first term in the rule of three direct; and multiply 810 by 12, the

product

product is 9720 for the third term. Then ſay, As 12960 is to 100 l. ſo is 9720 to the anſwer, *viz.* 75 l. as before. But if the terms had been placed after the ſecond order, *viz.* as in the margin; then the inverſe proportion is found in the loweſt numbers; and having compoſed the numbers for a ſingle rule of three, as in the ſecond rule foregoing, then the anſwer muſt be found by a ſingle rule of three inverſe: for here it falls out to multiply 810 by 12 for the firſt number, and 1440 by 9 for the third number; and then you muſt ſay, As 9720 is to 100 l. ſo is 12960 to the anſwer; which by inverſe proportion will be found to be 75 l. as before.

l.		*l.*		*l. s. d.*
6	:	100	: :	3—7—6
m.				*m.*
12				9

The queſtions in the 12th and 13th chapters may ſerve for thy farther experience.

CHAP. XV.

Single Fellowſhip.

1. FEllowſhip is that rule of plural proportion, whereby we balance accounts depending between divers perſons having put together a general ſtock, ſo that they may every man have his proportional part of gain, or ſuſtain his proportional part of loſs.

2. The rule of fellowſhip, is either ſingle, or it is double.

3. The ſingle rule is, when the ſtocks propounded are ſingle numbers, without any reſpect or relation to time, each partner continuing his money in ſtock for the ſame time.

4. In the ſingle rule of fellowſhip, the proportion is, As the whole ſtock of all the partners is in proportion to the total gain or loſs, ſo is each man's particular ſhare in the ſtock to his particular ſhare in the gain or loſs. Therefore take the total of all the ſtocks for the firſt term in the rule of three, and the whole gain or loſs for the ſecond term, and the particular ſtock of any one of the

the partners for the third term; then multiply and divide according to the ſeventh rule of the 9th chapter; and the fourth proportional number is the particular loſs or gain of him whoſe ſtock you made your third number: wherefore repeat the rule of three as often as there are particular ſtocks or partners in the queſtion; and the fourth terms produced upon the ſeveral operations, are the reſpective gain or loſs of thoſe particular ſtocks given; as in the examples following.

Queſt. 1. Two perſons, *viz.* A and B, bought a tun of wine for 20 l. of which A paid 12 l. and B paid 8 l. and they gained in the ſale thereof 5 l.; now I demand each man's ſhare in the gain according to his ſtock?

Firſt, I find the ſum of all their ſtocks, by adding them together, *viz.* 12 l. and 8 l. which are 20 l.; then, according to this rule, I ſay, firſt, If 20 l. (the ſum of their ſtock) require 5 l. (the total gain), how much will 12 l. (the ſtock of A) require? Multiply and divide by the ſeventh rule of the 9th chapter, and the anſwer is 3 l. for the ſhare of A in the gain. Then again I ſay, If 20 l. require 5 l. what will 8 l. require? The anſwer is 2 l. which is the gain of B. So I conclude, that the ſhare of A in the gain is 3 l. and the ſhare of B in the gain is 2 l. which in all is 5 l.

```
12
 8
——
20
```

```
     l.    l.      l.
If  20  :  5  ::  12
           12
         ————
     20)  60  (3 l.
          60
         ————
          (0)
```

```
     l.    l.      l.
If  20  :  5  ::  8
           8
         ————
     20)  40  (2 l.
```

Queſt. 2. Three merchants, *viz.* A, B, and C, enter upon a joint adventure; A put into the common ſtock 78 l. B put in 117 l. and C put in 234 l.; and they find, when they make up their accounts, that they have gained

gained in all 264 l.; now I desire to know each man's particular share in the gain?

First, I add their particular stocks together, and their sum is 429 l.; then I say, If 429 l. gain 264 l. what will 78 l. gain? and what will 117 l. and what will 234 l. (the stocks of A, B, and C) gain? Work by three several rules of three, and you will find that

	l.
	78
	117
	234
Sum	429

The gain of A, B, C is l. 48, 72, 144

The gain of		is	l.
	A		48
	B		72
	C		144
		Sum	264

Quest. 3. Four partners, *viz.* A, B, C, and D. built a ship, which cost 1730 l. of which A paid 346 l. B 519 l. C 692 l. and D 173 l. and her freight for a certain voyage is 370 l. which is due to the owners or builders; I demand each man's share therein according to his charge in building her?

Answer		l.
	A	74
	B	111
	C	148
	D	37
	Sum	370

Quest. 4. A, B, and C, enter into partnership for a certain time; A put into the common stock 364 l. B put in 482 l. C put in 500 l. and they gained 867; now I demand each man's share in the gain proportionable to his stock?

Answer		l.	s.	d.	Rem.
	A	234	9	3	354
	B	310	9	5	62
	C	322	1	3	930
	Sum	867	0	0	1346

The proof of the rule of ſingle fellowſhip.

5. To prove the rule of ſingle fellowſhip, add each man's particular gain or loſs together; and if the total ſum is equal to the general gain or loſs, then is the work rightly performed; but otherwiſe it is erroneous. Example: In the firſt queſtion of this chapter, the anſwer was, That the gain of A was 3 l. and the gain of B 2 l. which added together, makes 5 l. equal to the total gain given.

If in finding out the particular ſhares of the ſeveral partners any thing remain after diviſion is ended, ſuch remainders muſt be added together, (they being all fractions of the ſame denomination), and their ſum divided by the common diviſor in each queſtion, (*viz.* the total ſtock), and the quotient added to the particular gains; and then if the total ſum is equal to the total gain, the work is right, otherwiſe not.

As in the fourth queſtion, the remainders were 354, 62, and 930; which added together make 1346; which divided by 1346, (the ſum of their ſtocks), the quotient is 1 d.; which I add to the pence, *&c.* and the ſum of their ſhare is 867 l. equal to the total gain: wherefore I conclude the work is right.

C H A P. XVI.

Double Fellowſhip.

1. DOuble fellowſhip is, when ſeveral perſons enter into partnerſhip for unequal time; that is, when every man's particular ſtock hath relation to a particular time.

2. In the double rule of fellowſhip, multiply each particular ſtock by its reſpective time: and having added the ſeveral products together, make their ſum the firſt number or term in the rule of three, and the total gain or loſs the ſecond number, and the product of any one's particular ſtock by his time, the third term, and the fourth number in

in proportion thereunto is his particular gain or loſs whoſe product of ſtock and time is your third number.

Then repeat, as in ſingle fellowſhip, the rule of three, as often as there are products or partners; and the fourth terms thereby invented, are the numbers required.

Examples.

Queſt. 1. A and B enter into partnerſhip; A put in 40 l. for 3 months, B put in 75 l. for 4 months, and they gained 70 l.; now I demand each man's ſhare in the gain proportional to his ſtock and time? *Anſ.* A 20 l. B 50 l.

To reſolve this queſtion, I firſt multiply the ſtock of A, *viz.* 40 l. by its time, 3 months, and the product is 120. Then I multiply the ſtock of B by its time, *viz.* 75 l. by 4, and it produceth 300; which I add to the product of A, his ſtock, and time, and the ſum is 420 l. Then, by the rule of three direct, I ſay, As 420 (the ſum of the products) is to 70 (the total gain), ſo is 120 (the product of A his ſtock and time) to 20 l. (the ſhare of A in the gain.) Then I ſay again, As 420 is to 70, ſo is 300 to 50 l. (the ſhare of B in the gain.) And ſo much ought each to have for his ſhare.

	l.		*l.*
	40		75
	3		4
A	120	B	300
			120
		Sum	420

Queſt. 2. A, B, and C, make a ſtock for 12 months; A put in at firſt 364 l. and 4 months after that he put in 40 l.; B put in at firſt 408 l. and at the end of 7 months he took out 86 l.; C put in at firſt 148 l. and 3 months after he put in 86 l. more, and 5 months after that he put in 100 l. more; and at the end of 12 months their gain is found to be 1436 l. I deſire to know each man's ſhare in the gain according to his ſtock and time?

Firſt, I conſider, that the whole time of their partnerſhip is 12 months; then I proceed to find out the ſeveral products of ſtock and time, as followeth.

A had at firſt 364 l. for 4 months, wherefore their product is, — — — — — 1456

Then he put in 40 l. which, with the firſt ſum, makes 404 l. which continued the remainder of the time, *viz.* 8 months, and their product is, 3232

The ſum of the products of the ſtock and time of A is, — — — — — 4688

B had 408 l. in 7 months, whoſe product is 2856

And then took out 86 l.; therefore he left in ſtock 322 l. which continued the reſt of the time, *viz.* 5 months, whoſe product is, — — 1610

The ſum of the products of the ſtock and time of B is, — — — — — 4466

C put in 148 l. for 3 months, whoſe product is 444

Then he put in 86 l. which added to the firſt, *viz.* 148 l. makes 234 l. which lay in ſtock 5 months, their product is — — — 1170

Then he put in 100 l. more; ſo then he had in ſtock 334 l. which continued the remainder of the time, *viz.* 4 months, which multiplied together, produce — — — — 1336

The ſum of the product of the money and time
of C is, — — — — 2950
of B is, — — — — 4466
of A is, — — — — 4688

The total ſum of all the products is, — 12104

Then I ſay, As 12104 is to 1436, the total gain; ſo is 2950 to the ſhare of C in the total gain, *&c.* Go on as in the foregoing examples, and you will find their ſhares in the gain to be as followeth, *viz.*

Anſwer,

		l.	*s.*	*d.*	*Rem.*
The ſhare of	A	556	3	6	6192
	B	529	16	9	5496
	C	349	19	8	416
	Sum	1436	0	0	12104

Queſt.

Quest. 3. Three grasiers, A B, and C, take a piece of ground for 46 l. 10 s. in which A put 12 oxen for 8 months, B put in 16 oxen for 5 months, and C put in 18 oxen for 4 months; now the question is, What each man shall pay of the 46 l. 10s. for his share in that charge?

Answer.

		l.	s.
A	shall pay	18	0
B		15	0
C		13	10
		46	10

3. The proof of this rule is the same with that of single fellowship, laid down in the fifth rule of the 15th chapter. And note, that

If a loss be sustained instead of gain amongst partners, every man's share to be borne in the loss is to be found after the same method as their gain, whether their stocks be for equal or unequal time.

CHAP. XVII.

Alligation Medial.

1. THE rule of alligation is that rule in plural proportion by which we resolve questions wherein is a composition or mixture of divers simples; as also it is useful in composition of medicines, both for quantity, quality, and price. And its species are two, *viz.* medial and alternate.

2. Alligation medial is, when having the several quantities and prices of several simples propounded, we discover the mean price or rate of any quantity of the mixture compounded of those simples. And the proportion is,

As the sum of the simples to be mingled is to the total value of all the simples, so is any part or quantity of the composition or mixture to its mean rate or price.

Quest. 1. A farmer mingled 20 bushels of wheat, at 5 s. *per* bushel, and 36 bushels of rye, at 3 s. *per* bushel, with 40 bushels of barley, at 2 s. *per* bushel; now I desire to know what one bushel of that mixture is worth?

To resolve this question, add together the given quantities, and also their values, which is 96 bushels, whose

total value is 14 l. 8 s. as appeareth by the work following. For,

Bushels.	*l.*	*s.*
20 of wheat at 5 s. *per* bushel, is	5	0
36 of rye at 3 s. *per* bushel, is	5	8
40 of barley at 2 s. *per* bushel, is	4	0
The sum of their given quantities is, 96, and their value is,	14	8

Then say, by the rule of three direct, If 96 bushels cost (or is worth) 14 l. 8 s. what is 1 bushel worth?

Bush.		*l.*	*s.*		*Bush.*
96	:	14	8	: :	1
		20			
96)		288 (3 s.			
		288 *Facit* 3 s. *per* bush.			
		(0)			

Quest. 2. A vintner mingleth 15 gallons of Canary, at 8 s. *per* gallon, with 20 gallons of Malaga, at 7 s. 4 d. *per* gallon, with 10 gallons of Malaga, at 6 s. 10 d. *per* gallon, and 24 gallons of white-wine, at 4 s. *per* gallon; now I demand what a gallon of this mixture is worth? Work as in the last question, and you will find the answer to be 6 s. 2 d. $3\frac{57}{69}$ qrs.

Quest. 3. A grocer mingled 3 C. of sugar at 56 s. *per* C. with 3 C. of sugar at 3 l. 14 s. 8 d. *per* C. and with 6 C. at 1 l. 17 s. 4 d. *per* C. I desire to know the price of a hundred weight of that mixture? *Ans.* 2 l. 11 s. 4 d.

The proof of alligation medial.

3. The proof of this operation is, by the price of any quantity of the mixture, to find out the total value of the whole composition; and if it is equal to the total value of the several simples, the work is right; otherwise not: As in the first example, the answer to the question was, That 3 s. is the price of 1 bushel; wherefore I say, by the rule of proportion, If 1 bushel be 3 s. what is 96 bushels?

bushels? *Ans.* 14 l. 8 d. which is the total value of the several simples. Wherefore the work is right.

CHAP. XVIII.

Alligation Alternate.

1. ALligation alternate is, when there are given the particular prices of several simples, and thereby we discover such quantities of those simples, as being mingled together, shall bear a certain rate propounded.

2. When such a question is stated, place the given prices of the simples one over the other, and the propounded price of the composition against them, in such sort that it may represent a root, and they as so many branches springing from it; as in the following example.

Quest. 1. A certain farmer is desirous to mix 20 bushels of wheat at 5 s. or 60 d. *per* bushel; with rye at 3 s. or 36 d. *per* bushel, and with barley at 2 s. or 24 d. *per* bushel, and oats at 1 s. 6 d. *per* bushel; and desireth to mix such a quantity of rye, barley, and oats with the 20 bushels of wheat, as that the whole composition may be worth 2 s. 8 d. or 32 d. *per* bushel.

The prices of the simples being placed according to the last rule, with the price of the composition propounded as a root to them, will stand as on the margin.

$$32\left\{\begin{array}{l}60 \text{ pence}\\36\\24\\18\end{array}\right.$$

3. Having thus placed the given numbers, you are to link or combine the several rates of the simples the one to the other, by certain arches, in such sort, that one that is less than the root, or mean rate, may be linked or coupled to another that is greater than the mean rate. So the question last propounded will stand.

1. Thus,

$$32\left\{\begin{array}{l}60\\36\\24\\18\end{array}\right.$$

2. Or thus,

$$32\left\{\begin{array}{l}60\\36\\24\\18\end{array}\right.$$

3. Or thus,

$$32\left\{\begin{array}{l}60\\36\\24\\18\end{array}\right.$$

4. Then

4. Then take the difference between the root and the ſeveral branches, and place the difference of each againſt the number or branch with which it is coupled or linked; and having taken all the differences, and placed them as aforeſaid, then thoſe differences ſo placed, will ſhew you the number of each ſimple to be taken to make a compoſition to bear the mean rate propounded.

So the branches of the laſt queſtion being linked together, as in the firſt manner, I ſay, The difference between 32 and 60 is 28; which I put againſt 18, becauſe 60 is linked with 18. Then the difference between 32 and 36 is 4; which I put againſt 24, becauſe 36 is linked or coupled with 24. Then I ſay, The difference between 32 and 24 is 8; which I place againſt 36, for the reaſon aforeſaid. Then I ſay, The difference between 32 and 18 is 14; which I place againſt 60. And then the work will ſtand as you ſee in the margin.

$$32\left\{\begin{array}{l} 60 \\ 36 \\ 24 \\ 18 \end{array}\right. \quad \left|\begin{array}{r} 14 \\ 8 \\ 4 \\ 28 \end{array}\right.$$

So I conclude, that a compoſition made of 14 buſhels of wheat at 60 d. *per* buſhel, and 8 buſhels of rye at 36 d. *per* buſhel, and 4 buſhels of barley at 24 d. *per* buſhel, and 28 buſhels of oats at 18 d. *per* buſhel, will bear the mean price of 32 d. or 2 s. 8 d. *per* buſhel. And here obſerve, that in the compoſition there is but 14 buſhels of wheat; but I would mingle 20 buſhels. And this kind, or rather caſe, of alligation alternate, *viz.* when there is given a certain quantity of one of the ſimples, and the quantities of the reſt ſought to mingle with this given quantity, that the whole may bear a price propounded, is called *alternation partial.*

And the proportion to find out the ſeveral quantities to be mingled with the given quantity, is as followeth, *viz.*

As the difference annexed to the branch, that is the value of an integer of the given quantity, is to the other particular differences, ſo is the quantity given to the ſeveral quantities required.

So here, to find out how much rye, barley, and oats, muſt be mingled with the 20 buſhels of wheat, I ſay, by the ſingle rule of three direct, If 14 buſhels of wheat require

require 8 bushels of rye, what will 20 bushels of wheat require? *Ans.* $11\frac{6}{14}$ bushels of rye.

Again, If 14 bushels of wheat require 4 bushels of barley, what will 20 bushels of wheat require? *Ans.* $5\frac{10}{14}$ bushels of barley. Again, I say, If 14 bushels of wheat require 28 bushels of oats, what will 20 bushels of wheat require? *Ans.* 40 bushels of oats.

And now I say, That 20 bushels of wheat mingled with $11\frac{6}{14}$ bushels of rye, and $5\frac{10}{14}$ bushels of barley, and 40 bushels of oats, each bearing the rate as aforesaid, will make a composition or heap of corn that may yield 32 d. *per* bushel.

But if the branches had been coupled according to the second order or manner, the differences would have been thus placed, *viz.* the difference between 32 and 60 is 28; which I set against 24, because 60 is linked thereto: and the difference between 32 and 36 is 4; which I set against 18: and the difference between 32 and 24 is 8; which I set against 60: then the difference between 32 and 18 is 14; which I set against its yoke-fellow 36: and then I conclude, that if you mix 8 bushels of wheat with 14 bushels of rye, 28 bushels of barley, and 4 bushels of oats, each bearing the aforesaid prices, the whole mixture may be sold for 32 d. *per* bushel; as by the work in the margin.

$$32\left\{\begin{array}{l}60\\36\\24\\18\end{array}\right. \quad \left|\begin{array}{r}8\\14\\28\\4\end{array}\right.$$

You see by this work we have found how many bushels of rye, barley, and oats, ought to be mixed with 8 bushels of wheat. And to find out how many of each ought to be mixed with 20 bushels of wheat, I say, As 8 is to 14, so is 20 to 35 bushels of rye; as 8 is to 28, so is 20 to 70 bushels of barley; as 8 is to 4, so is 20 to 10 bushels of oats: whereby I conclude, that if to 20 bushels of wheat I put 35 bushels of rye, 70 bushels of barley, and 10 bushels of oats, bearing each the aforesaid price *per* bushel, that then a bushel of this mixture will be worth 32 d. or 2 s. 8 d.

And if the branches had been linked as you see in the third place, where each branch bigger than the root is linked to two that are less than the root; then in this case you must have placed the several differences between the root

root and branches, againſt thoſe two with which each is coupled: As firſt, the difference between 32 and 60 is 28; which I ſet againſt 24 and 18, becauſe it is coupled with them both. Then the difference between 32 and 36 is 4; which I ſet likewiſe againſt 24 and 18, becauſe 36 is linked to them both. Then the difference between 32 and 24 is 8; which I put againſt 60 and 36, becauſe 24 is linked to them both. Then the difference between 32 and 18 is 14; which I put againſt 60 and 36, the yoke-fellows of 18.

32	60	8, 14	22
	36	8, 14	22
	24	28, 4	32
	18	28, 4	32

Laſtly, I draw a line behind the differences, and add the differences which ſtand againſt each branch, and put the ſum behind the ſaid line, againſt its proper branch; as you ſee in the margin.

And now, by this work, I find, that 22 buſhels of wheat mingled with 22 buſhels of rye, and 32 buſhels of barley, and 32 buſhels of oats, each bearing the ſaid price, will make a mixture bearing the mean rate of 32 d. *per* buſhel.

And to find how much of each of the reſt muſt be mingled with 20 buſhels of wheat, I ſay,

As 22 is to 22, ſo is 20 to 20 buſhels of rye; as 22 is to 32, ſo is 20 to $29\frac{2}{22}$ buſhels of barley; as 22 is to 32, ſo is 20 to $29\frac{2}{22}$ buſhels of oats.

Whereby you ſee that queſtions of alligation alternate will admit of more true anſwers than one; for we have found three ſeveral anſwers to this firſt queſtion.

The proof of alternation partial.

Queſtions of alternation partial, are proved the ſame way with queſtions in alligation medial; which you may ſee in the third rule of the 17th chapter.

Queſt. 2. A grocer hath four ſorts of ſugar, *viz.* of 12 d. *per* lb. of 10 d. *per* lb. of 6 d. *per* lb. and of 4 d. *per* lb. and he would have a compoſition worth 8 d. *per* lb. the whole quantity whereof ſhould contain 144 lb. made of theſe four ſorts; I demand how much of each he muſt take?

Queſtions

Questions of this nature are resolved by that part of alligation alternate, called by arithmeticians, *alternation total, viz.* where there is given the sum and prices of several simples, to find out how much of each simple ought to be taken to make the said sum or quantity, so that it may bear a certain rate propounded.

To resolve this question, I place the several prices of the simples and mean rate propounded, and link them together as is directed in the second and third rules of this chapter, and place the differences between the root and branches, according to the fourth rule of this chapter; which will then stand one of these three ways, *viz.*

First.			Second.			Third.			
8	12	4	8	12	2	8	12	2, 4	6
	10	2		10	4		10	2, 4	6
	6	2		6	4		6	4, 2	6
	4	4		4	2		4	4, 2	6
		12			12				24

5. Then add the several differences together, which I have done, and the sums of the first and second order are 12 l. and of the third 24 lb. as you may see above. But it is required that there should be 144 lb. of the composition; therefore, to find the quantity of each simple to make the whole composition 144 lb. observe this general rule, *viz.*

As the sum of the differences is to the several differences, so is the total quantity of the composition to the quantity of each simple.

So to find how much of each sort of sugar I ought to take to make 144 lb. at 8 d. *per* lb. I say,

As 12 is to 4, so is 144 to 48 lb. at 12 d. *per* lb.
As 12 is to 2, so is 144 to 24 lb. at 10 d. *per* lb.
As 12 is to 2, so is 144 to 24 lb. at 6 d. *per* lb.
As 12 is to 4, so is 144 to 48 lb. at 4 d. *per* lb.

Whereby I find, that 48 lb. at 12 d. *per* lb. and 24 lb. at 10 d. *per* lb. and 24 lb. at 6 d. *per* lb. and 48 lb. at 4 d. *per* lb. will make a composition of sugar containing 144 lb. worth 8 d. *per* lb.

But

But as the branches are linked in the ſecond order, the anſwer will be 24 lb. at 12 d. *per* lb. and 48 lb. at 10 d. *per* lb. and 48 lb. at 6 d. *per* lb. and 24 lb. at 4 d. *per* lb. to make the ſaid quantity, and to bear the ſaid price.

And if you had worked as the branches are linked in the third order, then you would have found the quantity of each to have been 36 lb.

Queſt. 3. A vintner hath four ſorts of wine, *viz.* Canary at 10 s. *per* gallon, Malaga at 8 s. *per* gallon, Rheniſh wine at 6 s. *per* gallon, and white-wine at 4 s. *per* gallon, and he is minded to make a compoſition of them all of 60 gallons, that may be worth 5 s. *per* gallon; I deſire to know how much of each he muſt have?

The number of terms being ranked according to the ſecond rule of this chapter, the branches will be linked as followeth; but will admit of no other manner of coupling, becauſe there is but one branch that is leſs than the root; therefore all the reſt muſt be linked unto it; and the differences between the root and the three firſt branches, *viz.* 10, 8, and 6, which are 5, 3, and 1, muſt be ſet againſt 4, becauſe they are coupled with it; and the difference between the root, *viz.* 5, and 4, which is 1, muſt be ſet againſt the three other, becauſe it is linked to them all: ſo I find 1 gallon of Canary, 1 gallon of Malaga, 1 gallon of Rheniſh wine, and 9 gallons of white-wine, prized as above, being mingled together, will be worth 5 s. *per* gallon, the ſum being 12 gallons. But there muſt be 60 gallons; wherefore I ſay,

5	10	1	1
	8	1	1
	6	1	1
	4	5, 3, 1.	9
			12

As 12 is to 1, ſo is 60 to 5 gallons of Canary.
As 12 is to 1, ſo is 60 to 5 gallons of Malaga.
As 12 is to 1, ſo is 60 to 5 gallons of Rheniſh.
As 12 is to 9, ſo is 60 to 45 gallons of white-wine.

So that 5 gallons of Canary, 5 gallons of Malaga, 5 gallons of Rheniſh, and 45 gallons of white-wine, mingled together, will be in all 60 gallons, worth 5 s. *per* gallon; which was required.

Queſt. 4. A goldſmith hath gold of four ſeveral ſorts of fineneſs, *viz.* of 24 careſts fine, and of 22 careſts fine, of

20 carects fine, and of 15 carects fine, (see chap. 2. def. 2. of this book), and he would mingle so much of each with alloy, that a mass of 28 ounces of gold, so mingled, may bear 17 carects fine; I demand how much of each he must take? The second and third rule of this chapter being observed, (instead of the alloy I put 0, because it bears no fineness, but it makes a branch in the operation), the terms may be alligated, and the differences taken, by any of these four ways following, *viz.*

First thus,

17	24	17	17
	22	2	2
	20	2, 17	19
	15	5, 3	8
	0	7, 3	10
		Sum	56

Secondly thus,

17	24	2	2
	22	2	2
	20	17	17
	15	7, 5	12
	0	3,	3
		Sum	36

Thirdly thus,

17	24	2	2
	22	2	2
	20	2, 17	19
	15	7, 5, 3	15
	0	3	3
		Sum	41

Fourthly thus,

17	24	2, 17	19
	22	2, 17	19
	20	2, 17	19
	15	7, 5, 3	15
	0	7, 5, 3	15
		Sum	87

More ways may be given for the alligating or linking of the terms in this question, but these are sufficient for the industrious. And it shall also suffice to give an answer to the question, as the terms are linked the first way, not doubting but the ingenious practitioner will be able at his leisure to find answers to the other three ways, *viz.*

	oz.	*pw.*	*car.*
As 56 is to 17, so is 28 to	8	10	of 24
As 56 is to 2, so is 28 to	1	0	of 22
As 56 is to 19, so is 28 to	9	10	of 20
As 56 is to 8, so is 28 to	4	0	of 15
As 56 is to 10, so is 28 to	5	0	of alloy.

Thus much well practised and understood, is sufficient for understanding alligation.

N *The*

The proof of alternation total.

In queſtions of alternation total, the anſwer is given true, when the ſum or quantity of the ſimples found agrees with the ſum or quantity propounded; as in the laſt queſtion, the anſwer was 8 oz. 10 pw. of 24 carects fine, 1 oz. of 22 carects fine, 9 oz. 10 pw. of 20 carects fine, 4 oz. of 15 carects fine, and 5 oz. of alloy; which added together, makes 28 oz. the quantity propounded.

C H A P. XIX.

Reduction of Vulgar Fractions.

1. WHat a vulgar fraction is, and its parts and ſeveral kinds, hath been already ſhewed in the 19th, 20th, 21ſt, 22d, 23d, 24th, and 31ſt definitions of the firſt chapter of this book; which the learner is deſired diligently to obſerve before he proceeds.

2. How to reduce a vulgar fraction, we ſhall teach under theſe eight ſeveral heads, or rules, following, *viz.*

1. To reduce a mixed number into an improper fraction.

2. To reduce a whole number into an improper fraction.

3. To reduce an improper fraction into its equivalent whole or mixed number.

4. To reduce a fraction into its loweſt terms equivalent to the fraction given.

5. To find the value of a fraction in the known parts of coin, weight, meaſure, *&c.*

6. To reduce a compound fraction to a ſimple one of the ſame value.

7. To reduce divers fractions having unequal denominators, to fractions of the ſame value having equal denominators.

8. To reduce a fraction of one denomination to another of the ſame value.

I. *To reduce a mixed number to an improper fraction.*

The rule is, Multiply the integer-part, or whole number by the denominator of the fraction, and to the product add the numerator, and that ſum place over the denominator for a new numerator; ſo this new fraction ſhall be equal to the mixed

mixed number given. *Vid. cap.* 1. *defin.* 31. and 23. As for example:

1. Reduce $18\frac{3}{7}$ into an improper fraction. Multiply the whole number 18 by 7 the denominator, and to the product add the numerator 3, the sum is 129; which put over the denominator 7, and it makes $\frac{129}{7}$ for the answer, as on the margin.

$$\begin{array}{r} 18\frac{3}{7} \\ 7 \\ \hline 129 \end{array}\quad \textit{Facit } \frac{129}{7}$$

2. Reduce $183\frac{5}{12}$ to an improper fraction. *Facit* $\frac{2201}{12}$

3. Reduce $50\frac{13}{21}$ to an improper fraction. *Facit* $\frac{1063}{21}$

II *To reduce a whole number into an improper fraction.*

The rule is, Multiply the given number by the intended denominator, and place the product for the numerator over it. As for example:

1. Let it be required to reduce 15 into a fraction, whose denominator shall be 12. To effect which, I multiply 15 by the intended denominator 12, the product is 180; which I place over 12 as a numerator, and it makes $\frac{180}{12}$, which is equal to 15, as was required, as *per* margin.

$$\begin{array}{r} 15 \\ 12 \\ \hline 30 \\ 15 \\ \hline 180 \end{array}\quad \textit{Facit } \frac{180}{12}$$

2. Reduce 36 into an improper fraction, whose denominator shall be 26. *Facit* $\frac{936}{26}$.

3. Reduce 135 into an improper fraction, whose denominator shall be 16. *Facit* $\frac{2160}{16}$.

III. *To reduce an improper fraction into its equivalent whole or mixed number.*

The rule is, Divide the numerator by the denominator, and the quotient is the whole number equal to the fraction; and if any thing remain, put it for a numerator over the divisor. As for example:

1. Reduce $\frac{436}{8}$ into its equivalent mixed number. Divide the numerator 436 by the denominator 8, and the quotient is 54, and 4 remains; which put for a numerator over the divisor 8, the answer is $54\frac{4}{8}$; as in the margin.

$$\begin{array}{rl} 8)\ 436 & (54 \\ 40 & \\ \hline 36 & \textit{Fac. } 54\frac{4}{8} \\ 32 & \\ \hline (4) & \end{array}$$

2. Reduce $\frac{1476}{15}$ to a mixed number. *Facit* $98\frac{6}{15}$.
3. Reduce $\frac{2357}{136}$ to a mixed number. *Facit* $173\frac{48}{136}$.

IV. *To reduce a fraction into its lowest terms equivalent to the fraction given.*

The rule is, 1. If the numerator and denominator are even numbers, take half the one and half the other, as often as may be; and when either of them falls out to be an odd number, then divide them by any number that you can discover will divide both numerator and denominator without any remainder; and when you have thus proceeded as low as you can reduce them, then this new fraction so found out, shall be the fraction you desire, and will be in value equal to the given fraction.

Example 1. Let it be required to reduce $\frac{192}{336}$ into its lowest terms. First, I take the half of the numerator 192, and it is 96; then half of the denominator, and it is 168; so that now it is brought to $\frac{96}{168}$, and next to $\frac{48}{84}$, and by halving still, to $\frac{24}{42}$, and their half is $\frac{12}{21}$: and now I can no longer halve it, because 21 is an odd number; wherefore I try to divide them by 3, *&c.* and I find 3 divides them both without any remainder, and brings them to $\frac{4}{7}$, as *per* margin. So I conclude $\frac{4}{7}$ thus found, to be equal in value to the given fraction $\frac{192}{336}$.

$$\frac{192}{336}\Big|\frac{96}{168}\Big|\frac{48}{84}\Big|\frac{24}{42}\Big|\frac{12}{21}\Big|\frac{4}{7}$$

2. What is $\frac{1036}{1184}$ in its lowest terms? *Ans.* $\frac{7}{8}$.
3. What is $\frac{1342}{1586}$ in its lowest terms? *Ans.* $\frac{11}{13}$.

There is yet another way more excellent than the former, to reduce a fraction into its lowest terms; and that is, 2. by finding a common measurer, *viz.* the greatest number that will divide the numerator and denominator without any remainder, and by that means reduce a fraction to its lowest terms at the first work. And to find out this common measurer, divide the denominator by the numerator; and if any thing remains, divide your divisor thereby; and if any thing yet remain, then divide your last divisor by it. Do so until you find nothing remaining; then this last divisor shall be your greatest common measurer, which will divide both numerator and denominator, and reduce them both into their lowest terms at one work. *Vid. Ought. Cla. Math. cap.* 7.

Example

Example 4. Reduce $\frac{228}{304}$ into its lowest terms by a common measurer. To effect which, I divide the denominator 304, by the numerator 228, and there remains 76. Then I divide 228 (the first divisor) by 76 (the remainder), and it quotes 3, and nothing remains. Wherefore the last divisor 76 is the common measurer; by which I divide the numerator of the given fraction, *viz.* 228, it quotes 3 for a new numerator. Then I divide the denominator 304 by 76, and it quotes 4 for a new denominator. So that now I have found $\frac{3}{4}$ equal to $\frac{228}{304}$.

5. Reduce $\frac{6048}{8736}$ into its lowest terms by a common measurer. *Facit* $\frac{9}{13}$.

6. Reduce $\frac{1081}{10143}$ into its lowest terms by a common measurer. *Facit* $\frac{23}{83}$.

A compendium.

Note, That if the numerator and denominator of a fraction end each with a cipher or ciphers, then cut off as many ciphers from the one as from the other, and the remaining figures will be a fraction of the same value, thus, $\frac{1400}{1100}$ will be found to be reduced to $\frac{14}{11}$, by cutting off the two ciphers from the numerator and denominator with a dash of the pen, thus, $\frac{14|00}{11|00}$; and $\frac{460}{700}$, will be $\frac{46}{70}$, thus, $\frac{46|0}{70|0}$. *&c.*

V. *To find the value of a fraction in the known parts of coin, weight*, &c.

The rule is, Multiply the numerator by the parts of the next inferior denomination that are equal to an unit of the same denomination with the fraction, then divide that product by the denominator, and the quote gives you its value in the same parts you multiplied by: and if any thing remain, multiply it by the parts of the next inferior denomination, and divide as before. Do so till you can bring it no lower, and the several quotients will give you the value of the fraction required; and if any thing at last remain, place it for a numerator over the former denominator. Some few examples will make the rule plain.

1. What is the value of $\frac{27}{29}$ l. Sterling? To answer this question, I multiply the numerator 27 by 20 (the shillings in a pound), the product is 540; which I divide

by 29, (the denominator), and the quotient is 18 s. and there remains 18; which I multiply by 12, and the product (216) I divide by the denominator 29, the quotient is 7 d. and 13 remains; which I multiply by 4, the product is 52; which I still divide by 29, the quotient is 1 farthing, and there remaineth 23; which I put for a numerator over the denominator 29: so I find the value of $\frac{27}{29}$ l. to be 18 s. 7 d. 1 qr. $\frac{23}{29}$, as by the operation on the margin. And after the same manner are the values of the fractions in the several examples following found out.

		l.
	27	$\frac{27}{29}$
Multiply	20	
29)	540	(18 s. 7 d. 1 $\frac{23}{29}$ *qrs.*
	29	
	250	
	232	
Remains	18	
Multiply	12	
	36	
	18	
29)	216	(7 d.
	203	
Remains	13	
Multiply	4	qr.
29)	52	(1 $\frac{23}{29}$
	29	
Remains	23	

s. *d.* *qr.*
Facit 18—7—1 $\frac{23}{29}$.

2. What is the value of $\frac{11}{15}$ l. Sterling? *Facit* 14 s. 8 d.

3. What is the value of $\frac{28}{137}$ l. Sterling? *Facit* 4 s. 1 d. $\frac{7}{137}$.

4. What is $\frac{16}{21}$ C. weight? *Facit* 3 qrs. 1 lb. 5 oz. $\frac{7}{21}$.

5. What is $\frac{136}{371}$ lb. Troy weight? *Facit* 4 oz. 7 pw. 23 gr. $\frac{179}{371}$.

6. What is $\frac{41}{50}$ of a year? *Ans.* 299 days, 7 hours, 12 min.

VI. *To reduce a compound fraction to a simple one of the same value.*

What a compound fraction is hath been shewn in chap. 1. defin. 24; and to reduce it to a simple fraction of the same value,

The rule is, Multiply the numerators continually, and place the last product for a new numerator; then multiply

tiply the denominators continually, and place the last product for a new denominator: so this single fraction shall be equal to the compound fraction. For example:

1. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction.

Multiply the numerators 2, 3, and 5 together, they make 30 for a new numerator; then multiply the denominators 3, 5, and 8 together, and their product is 120 for a denominator; so the simple fraction is $\frac{30}{120}$, and cutting off the ciphers, it is $\frac{3}{12}$, equal to $\frac{1}{4}$ by the 4th rule; as in the margin.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 8 \\ \hline 120 \end{array} \qquad \begin{array}{r} 3 \\ 2 \\ \hline 6 \\ 5 \\ \hline 30 \end{array}$$

Facit $\frac{30}{120}$, or $\frac{3}{12}$, or $\frac{1}{4}$

2. What is $\frac{7}{10}$ of $\frac{5}{8}$ of $\frac{4}{7}$ of $\frac{11}{12}$? *Ans.* $\frac{1540}{6720}$, or $\frac{154}{672}$, or $\frac{77}{336}$ in its lowest terms.

3. What is $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{29}$? *Ans.* $\frac{1001}{2872}$.

By this you may know how to find the value of a compound fraction, *viz.* first reduce it to a simple one, and then find out its value by the fifth rule foregoing. As,

4. What is the value of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{12}$ of a pound? *Ans.* 9 s. 4 d. 2 qrs.

VII. *To reduce fractions of unequal denominators, to fractions of the same value having equal denominators.*

The rule is, Multiply all the denominators together, and the product shall be the common denominator. Then multiply each numerator into all the denominators, except its own, and the last product put for a numerator over the denominator, found out as before: so this new fraction is equal to that fraction whose numerator you multiplied into the said denominators. Do so by all the numerators given, and you have your desire. For example:

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator. Multiply the denominators 4, 5, 6, and 8, together continually; and put the product, 960, for the common denominator. Then multiply the numerator 3 into the denominators 5, 6, and 8, and the product is 720, which is a numerator to 960 (found as before). So $\frac{720}{960}$ is equal to the first fraction $\frac{3}{4}$. Then I proceed to find a new numerator to the second fraction, *viz.* $\frac{4}{5}$, and I mul-

tiply

tiply 4 into all the denominators except its own, *viz.* into 4, 6, and 8, which produceth $\frac{768}{960}$ equal to $\frac{4}{5}$. Then I multiply the numerator 5 into the denominators 4, 5, and 8; the product is $\frac{800}{960}$, equal to $\frac{5}{6}$. Then I multiply the numerator 7 into the denominators 4, 5, and 6; the product is $\frac{840}{960}$, equal to $\frac{7}{8}$: and the work is done. So that for $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$, I have $\frac{720}{960}$, $\frac{768}{960}$, $\frac{800}{960}$, and $\frac{840}{960}$.

2. Reduce $\frac{11}{12}$, $\frac{24}{25}$, and $\frac{19}{21}$, into a common denominator. *Faciunt* $\frac{5770}{6300}$, $\frac{6048}{6300}$, $\frac{5700}{6300}$.

VIII. *To reduce a fraction of one denomination to another.*

This is either aſcending or deſcending: aſcending, when a fraction of a ſmaller is brought to a greater denomination; deſcending, when a fraction of a greater denomination is brought to a lower.

1. When a fraction is to be brought from a leſſer to a greater denomination, then make of it a compound fraction, by comparing it with the intermediate denominations between it and that you would have it reduced to: then (by the 6th rule foregoing) reduce your compound to a ſimple fraction, and the work is done. For example:

Queſt. 1. It is required to know what part of a pound Sterling $\frac{5}{7}$ of a penny is?

To reſolve this queſtion, I conſider that 1 d. is $\frac{1}{12}$ of a ſhilling, and a ſhilling is $\frac{1}{20}$ of a pound; wherefore, $\frac{5}{7}$d. is $\frac{5}{7}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound; which, by the ſaid 6th rule, I find to be $\frac{5}{1680}$ of a pound Sterling, or Engliſh money.

Queſt. 2. What part of a pound Troy weight is $\frac{4}{5}$ of a penny weight? *Anſ.* $\frac{4}{5}$ of $\frac{1}{20}$ of $\frac{1}{12}$ lb. equal to $\frac{4}{1200}$ lb. Troy.

2. When a fraction is to be brought from a greater to a leſſer denomination, then multiply the numerator by the parts contained in the ſeveral denominations betwixt it and the parts you would reduce it to; then place the laſt product over the denominator of the given fraction. For example:

Queſt. 3. I would reduce $\frac{3}{5}$ l. to the fraction of a penny; to do which, I multiply the numerator 3 by 20 and 12, the product is 720; which I put over the denominator 5, it makes $\frac{720}{5}$ of a penny, equal to $\frac{3}{5}$ l.

Queſt. 4. What part of an ounce Troy is $\frac{5}{16}$ lb.? *Anſ.* $\frac{60}{16}$ oz.

CHAP.

CHAP. XX.

Addition of Vulgar Fractions.

1. IF your fractions to be added have a common denominator, then add all the numerators together, and place their ſum for a numerator to the common denominator, which new fraction is the ſum of all the given fractions; and if it be improper, reduce it to a whole or mixed number, by the 3d rule of the 19th chapter.

Queſt. 1. What is the ſum of $\frac{7}{24}$, $\frac{9}{24}$, $\frac{16}{24}$, and $\frac{14}{24}$?

The denominators are equal; *viz.* every one is 24; wherefore add the numerators together, *viz.* 7, 9, 16, and 14, their ſum is 46; which put over the denominator 24, it makes $\frac{46}{24}$, the ſum of the given fractions; which will be reduced to the mixed number $1\frac{22}{24}$, or $1\frac{11}{12}$.

2. But if the fractions to be added have unequal denominators, then reduce them to a common denominator by the 7th rule of the 19th chapter, and then add the numerators together, and put the ſum over the common denominator, *&c.* as before in the laſt example.

Queſt. 2. What is the ſum of $\frac{3}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$?

The fractions reduced to a common denominator are $\frac{2880}{4800}$, $\frac{4200}{4800}$, $\frac{4320}{4800}$, and $\frac{4400}{4800}$; the ſum of their numerators is 15800; which put over the common denominator, makes $\frac{15800}{4800}$, or $\frac{158}{48}$, equal to the mixed number $3\frac{14}{48}$, or $3\frac{7}{24}$, for the ſum required.

Queſt. 3. What is the ſum of $\frac{13}{17}$, $\frac{21}{49}$, and $\frac{39}{47}$? *Anſ.* $2\frac{903}{39151}$.

3. If you a:e to add mixed numbers together, then add the fractional parts as before; and if their ſum be an improper fraction, reduce it to a mixed number, and add its integral part to the integral parts of the given mixed numbers, and the work is done.

Queſt. 4. What is the ſum of $13\frac{3}{4}$ and $24\frac{5}{8}$?

Firſt, add the fractions $\frac{3}{4}$ and $\frac{5}{8}$, the ſum is $1\frac{12}{32}$; then add the integer 1 to 13 and 24, their ſum is 38; and put after it the fraction $\frac{12}{32}$, it is $38\frac{12}{32}$, for the anſwer, or it is $38\frac{3}{8}$.

Queſt. 5. What is the ſum of $48\frac{3}{7}$, $64\frac{5}{8}$, and $130\frac{3}{4}$? *Facit* $243\frac{180}{224}$, or $243\frac{45}{56}$.

4. If

4. If any of the fractions to be added is a compound fraction, it must first be reduced to a simple fraction by the 6th rule of chapter 19. and then add it to the rest according to the 2d rule of this chapter. For example:

Quest 6. What is the sum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{6}$?

Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{6}$ into a simple fraction, and it is $\frac{105}{192}$; which reduced with the other two, and added, are $2\frac{600}{4608}$.

Quest. 7. What is the sum of $\frac{11}{12}$, and $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$? *Ans.* $1\frac{5}{12}$.

5. If the fractions to be added are not of one denomination, they must be so reduced, and then proceed as before.

Quest. 8. What is the sum of $\frac{3}{4}$ l. and $\frac{5}{6}$ s.?

Of the given fractions here, one is of a pound, and the other the fraction of a shilling; and before you can add them together, you must reduce $\frac{5}{6}$ s. to the fraction of a pound, as the other is, (by the 8th rule of chapter 19.), and it makes $\frac{5}{120}$ l.; then $\frac{3}{4}$ l. and $\frac{5}{120}$ l. will be found to be $\frac{380}{480}$ l. or $\frac{38}{48}$ l. by the 7th rule of chapter 19. and in its lowest terms $\frac{19}{24}$ l. by the 4th rule of chapter 19.

It would have been the same, if (by the latter part of the 8th rule of chapter 19.) you had reduced $\frac{3}{4}$ l. to the fraction of a shilling; which you would have found to have been $\frac{60}{4}$ s.; which added to $\frac{5}{6}$ s. by the said 7th rule of the last chapter, the sum is $15\frac{20}{24}$ s. which is equal to the sum found, as before, *viz.* $\frac{19}{24}$ l.: for (by the 5th rule of chapter 19.) the value of $\frac{19}{24}$ l. will be found to be 15 s. 10 d.; and so will $15\frac{20}{24}$ s. be found to be just as much.

Quest. 9. What is the sum of $\frac{3}{4}$ l. $\frac{3}{4}$ s. and $\frac{3}{4}$ d.? *Ans.* $\frac{242880}{307200}$ l. or $\frac{24288}{30720}$ l. or in its lowest terms $\frac{253}{320}$ l.

C H A P. XXI.

Subtraction of Vulgar Fractions.

1. THE rules in addition for reducing the given fractions to one denomination, are here to be observed: for before subtraction can be made, the fractions must be reduced to a common denominator; then subtract one numerator from the other, and place the remainder

mainder over the common denominator; which fraction shall be the excess or difference between the given fractions. For example:

Quest. 1. What is the difference between $\frac{3}{4}$ and $\frac{5}{7}$? The given fractions are reduced to $\frac{21}{28}$ and $\frac{20}{28}$; then subtract the numerator 20 from the numerator 21, and there remains 1; which being put over the denominator 28, makes $\frac{1}{28}$ for the answer, or difference between $\frac{3}{4}$ and $\frac{5}{7}$.

Quest. 2. What is the difference between $\frac{5}{6}$ and $\frac{3}{7}$ of $\frac{5}{8}$?

Reduce the compound fraction $\frac{3}{7}$ of $\frac{5}{8}$ to a simple fraction; then proceed as before; and the answer is $\frac{29}{24}$, equal to $1\frac{5}{24}$.

2. When a fraction is given to be subtracted from a whole number, subtract the numerator from the denominator, and put the remainder for a numerator to the given denominator, and subtract an unit (for that you borrowed) from the whole number, and the remainder place before the fraction found, as before; which mixed number is the remainder or difference sought. For example:

Quest. 3. Subtract $\frac{7}{10}$ from 48.

Ans. 47 $\frac{3}{10}$: for if you subtract 7 (the numerator) from 10 (the denominator), there remains 3; which put over 10 is $\frac{3}{10}$, and 1 I borrowed from 48, rests 47, to which join $\frac{3}{10}$, and it makes $47\frac{3}{10}$ for the excess.

Quest. 4. Subtract $\frac{13}{21}$ from 57, remains $56\frac{8}{21}$.

3. If it be required to subtract a fraction from a mixed number, or one mixed number from another, reduce the fraction to a common denominator; and if the fraction to be subtracted be less than the other, then subtract the lesser numerator from the greater, and that is a numerator for the common denominator. Then subtract the lesser integral part from the greater, and the remainder with the remaining fractions thereto annexed is the difference required between the two given mixed numbers. For example:

Quest. 5. Subtract $26\frac{3}{7}$ from $54\frac{5}{6}$.

First, subtract $\frac{3}{7}$, *viz.* $\frac{18}{42}$ from $\frac{5}{6}$, *viz.* $\frac{35}{42}$, the remainder is $\frac{17}{42}$; then 26 from 54, remaineth 28; to which annex $\frac{17}{42}$, it makes $28\frac{17}{42}$ for the answer.

4. But if the fraction to be subtracted is greater than the fraction from whence you subtract, then having first reduced the fractions to a common denominator, take the

numerator

numerator of the greatest fraction out of the denominator, and add the remainder to the numerator of the lesser fraction, and their sum is a new numerator to the common denominator, which fraction note; then (for the 1 you borrowed) add 1 to the integral part to be subtracted, and subtract it from the greater number, and to the remainder annex the fraction you noted before; so this new mixed number shall be the difference sought. For example:

Quest. 6. Subtract $14\frac{3}{4}$ from $29\frac{4}{7}$.

The fractions reduced are $\frac{21}{28}$ and $\frac{16}{28}$, *viz.* $\frac{3}{4}$ equal to $\frac{21}{28}$, and $\frac{4}{7}$ equal to $\frac{16}{28}$: now, I should subtract $\frac{21}{28}$ from $\frac{16}{28}$, but I cannot; therefore I subtract 21 from 28, rests 7; which added to 16 (the lesser numerator), makes 23 for a numerator to 28, *viz.* $\frac{23}{28}$. Then I come to the integral parts 14 and 29; and say, 1 that I borrowed and 14 is 15, which taken from 29, there rests 14; to which annexing $\frac{23}{28}$ it is $14\frac{23}{28}$ for the remainder, or difference between $14\frac{3}{4}$ and $29\frac{4}{7}$.

Quest. 7. Subtract $36\frac{9}{10}$ from $74\frac{4}{9}$. *Facit* $37\frac{49}{90}$.

CHAP. XXII.

Multiplication of Vulgar Fractions.

1. IF the multiplicand and multiplier are simple or single fractions, then multiply the numerators together for a new numerator, and the denominators for a new denominator, and the new fraction is the product required.

Quest. 1. What is the product of $\frac{5}{7}$ by $\frac{9}{11}$? *Facit* $\frac{45}{77}$; for the numerators 5 and 9 being multiplied, make 45; and the denominators 7 and 11 being multiplied, make 77.

Quest. 2. What is the product of $\frac{18}{23}$ by $\frac{21}{37}$? *Facit* $\frac{378}{851}$.

2. If the fractions to be multiplied be mixed numbers, reduce them to improper fractions by the 1st rule of the 19th chapter; then proceed as before.

Quest. 3. What is the product of $48\frac{3}{5}$ by $13\frac{5}{6}$?

The given mixed numbers being reduced to improper fractions are $48\frac{3}{5}$ equal to $\frac{243}{5}$, and $13\frac{5}{6}$ equal to $\frac{83}{6}$: now $\frac{243}{5}$ multiplied by $\frac{83}{6}$, according to the 1st rule of this chapter, produceth $\frac{20169}{30}$, or $672\frac{9}{30}$.

Quest.

Quest. 4. What is the product of $430\frac{6}{10}$ by $18\frac{3}{7}$? *Facit* $\frac{555474}{70}$, or $7935\frac{24}{70}$.

3. If a compound fraction is to be multiplied by a simple fraction, first reduce the compound fraction into a simple fraction; then multiply the one by the other, as is taught above.

Quest. 5. What is the product of $\frac{16}{21}$ by $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$?

The compound fraction $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$ reduced is $\frac{60}{140}$, or $\frac{6}{14}$; which multiplied by $\frac{16}{21}$, produceth $\frac{96}{294}$; which in its lowest terms is $\frac{16}{49}$ for the answer.

And if the multiplicand and multiplier are both compound fractions, reduce them both to simple ones; then multiply these new fractions as before, so you have the product.

Quest. 6. What is the product of $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{1}{2}$? *Ans.* $\frac{18}{120}$, in its lowest terms $\frac{3}{20}$.

Quest. 7. What is the product of $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{2}{5}$ of $\frac{5}{6}$? *Ans.* $\frac{60}{360}$, or $\frac{6}{36}$, or in its lowest terms $\frac{1}{6}$.

4. If a fraction be to be multiplied by a whole number, put under the given whole number an unit for a denominator, whereby it will be an improper fraction; then multiply these fractions as before. For example:

Quest. 8. What is the product of 24 by $\frac{2}{3}$? *Ans.* $\frac{48}{3}$: for 24, by putting an unit under it, will be $\frac{24}{1}$; and $\frac{2}{3}$ multiplied by $\frac{24}{1}$, produceth $\frac{48}{3}$ or 16.

Quest. 9. What is the product of 36 by $\frac{9}{11}$? *Ans.* $\frac{324}{11}$, or $29\frac{5}{11}$.

CHAP. XXIII.

Division of Vulgar Fractions.

1. IF the dividend and the divisor are both simple fractions, then multiply the numerator of the dividend into the denominator of the divisor, and the product is a new numerator; and multiply the denominator of the dividend into the numerator of the divisor, and the product is a new denominator; which new fraction thus found, is the quotient you desire. For example:

Quest. 1. What is the quotient of $\frac{5}{8}$ divided by $\frac{3}{5}$? *Ans.* $\frac{25}{24}$, or $1\frac{1}{24}$. For first I multiply (5) the numerator of the divi-

$$\frac{3}{5}\Big)\frac{5}{8}\Big(\frac{25}{24}$$

 dend

dend into (5) the denominator of the divisor, and the product (25) is a numerator for the quotient. Then I multiply (8) the denominator of the dividend into (3) the numerator of the divisor, and the product (24) I put in the quotient for a denominator. So I find $\frac{25}{24}$ is the quotient sought.

Quest. 2. What is the quotient of $\frac{10}{21}$ divided by $\frac{2}{3}$? *Ans.* $\frac{30}{42}$, equal to $\frac{5}{7}$ in its lowest terms.

2. But if you would divide a simple fraction by a compound, or a compound by a simple, first reduce such compound to a simple fraction; then go on as before.

Quest. 3. What is the quotient of $\frac{3}{10}$ divided by $\frac{3}{4}$ of $\frac{2}{3}$? *Ans.* $\frac{36}{60}$, or $\frac{3}{5}$. First reduce $\frac{3}{4}$ of $\frac{2}{3}$ into a simple fraction, and it is $\frac{6}{12}$; by which $\frac{3}{10}$ being divided, the quotient is $\frac{36}{60}$, equal in its lowest terms to $\frac{3}{5}$.

And if the dividend and divisor be both compound fractions, reduce them both to simple fractions; then divide the one by the other, as in rule 1. foregoing.

Quest. 4. What is the quote of $\frac{2}{3}$ of $\frac{3}{4}$ divided by $\frac{2}{3}$ of $\frac{5}{6}$? *Ans.* $\frac{180}{120}$, or $\frac{18}{12}$, or $1\frac{6}{12}$, or $1\frac{1}{2}$ in its lowest terms.

3. If the dividend, or divisor, or both, are mixed numbers, reduce them to improper fractions, and perform division as you were taught before.

Quest. 5. What is the quote of $12\frac{3}{4}$ divided by $21\frac{4}{5}$? *Ans.* $\frac{255}{436}$: for $12\frac{3}{4}$ is equal to $\frac{51}{4}$, and $21\frac{4}{5}$ is equal to $\frac{109}{5}$; and the quote of $\frac{51}{4}$, divided by $\frac{109}{5}$, is, as before, $\frac{255}{436}$.

4. If you divide a fraction by a whole number, or a whole number by a fraction, make the whole number an improper fraction, by putting an unit for a denominator to it, as was taught in rule 4. of chapter 22. and then perform division as was before taught. For example:

Quest. 6. What is the quote of 8 divided by $\frac{3}{5}$? *Ans.* $\frac{40}{3}$, which is equal to $13\frac{1}{3}$, being reduced as is before directed. See the work in the margin.

$$\frac{3}{5}\Big)\frac{8}{1}\Big(\frac{40}{3} \text{ or } 13\tfrac{1}{3}$$

Quest. 7. What is the quotient of $\frac{3}{5}$ divided by 8? *Ans.* $\frac{3}{40}$, as *per* margin.

$$\frac{8}{1}\Big)\frac{3}{5}\Big(\frac{3}{40}$$

CHAP.

C H A P. XXIV.

The Rule of Three direct in Vulgar Fractions.

1. AS in the rule of three in whole numbers, ſo likewiſe in fractions, you muſt ſee that the fractions of the firſt and third places be of the ſame denomination.

2. See, if any of the given fractions be compound, that they be reduced to ſimple ones of the ſame value.

3. If there are given mixed numbers, reduce them to improper fractions by the 1ſt rule of chapter 19.

4. If any of the three terms is a whole number, make it an improper fraction, by conſtituting an unit for its denominator.

Having reduced your fractions as is directed in the four laſt rules, then proceed to a reſolution, which is performed the ſame way as in whole numbers, reſpect being had to the rules delivered for the working of fractions, *viz.* Multiply the ſecond and third fractions together, according to the rules of chapter 22. and divide the product by the firſt fraction, according to the rules of chapter 23. and the quotient is the anſwer. Or, which is better,

5. Multiply the numerator of the firſt fraction into the denominators of the ſecond and third, and the product is a new denominator; then multiply the denominator of the firſt fraction into the numerators of the ſecond and third, and the product is a new numerator; which new fraction is the fourth proportional or anſwer; which, if it be an improper fraction, muſt be reduced to a whole or mixed number by the 3d rule of chapter 19. For example:

Queſt. 1. If $\frac{3}{4}$ yard of cloth coſt $\frac{5}{8}$ l. what will $\frac{9}{10}$ yard coſt?

Having placed the given fractions according to the 6th rule of chapter 10. I proceed to the reſolution. And firſt I multiply the numerator of the firſt fraction (3) into 8 and 10, the denominators of the ſecond and third fractions, and the product is 240 for a denominator; then I

Yards.	*l.*		*Yards.*		*l.*
$\frac{3}{4}$:	$\frac{5}{8}$	: :	$\frac{9}{10}$	:	$\frac{180}{240}$

Facit $\frac{180}{240}$, equal to $\frac{3}{4}$ l. or 15 s.

 multiply

multiply 4 the denominator of the first fraction into 5 and 9, the numerators of the second and third fractions, the product is 180 for a numerator; which numerator 180, and denominator 240, make $\frac{180}{240}$ l. for the answer, equal to $\frac{3}{4}$ l. or 15 s.

Quest. 2. If $\frac{2}{3}$ l. buy $\frac{5}{6}$ yard of cloth, what will $\frac{11}{12}$ yard cost at that rate? *Ans.* $\frac{132}{180}$ l. equal to $\frac{11}{15}$ l. or 14 s. 8 d.

Quest. 3. If $\frac{7}{8}$ lb. cost $\frac{3}{4}$ s. what will $\frac{8}{9}$ s. buy? *Ans.* $\frac{224}{210}$ lb. equal to $1\frac{1}{27}$ lb.

Quest. 4. If $\frac{3}{5}$ of an ell of holland cost $\frac{1}{3}$ of a pound, how much will $12\frac{2}{3}$ ells cost at that rate? *Ans.* $\frac{190}{27}$ l. equal to $7\frac{1}{27}$ l.

In resolving the last question and the two next, observe the third rule of this chapter.

Quest. 5. If $\frac{9}{10}$ C. cost 284 s. what will $7\frac{1}{2}$ C. cost at that rate? *Ans.* $239\frac{7}{12}$ s. or 11 l. 19 s. 7 d.

Quest. 6. If $3\frac{1}{4}$ yards of velvet cost $3\frac{5}{8}$ l. how much will $10\frac{1}{2}$ yards cost at that rate? *Ans.* $11\frac{37}{52}$ l.

Quest. 7. If 3 yards of broad cloth cost $2\frac{4}{5}$ l. what will $14\frac{3}{7}$ yards cost? *Ans.* 13 l. 9 s. 4 d.

In working the last question and the four next, observe the 4th rule of this chapter.

Quest. 8. If 14 lb. of pepper cost 14 s. $6\frac{3}{5}$ d. I demand the price of $73\frac{3}{4}$ lb.? *Ans.* 3 l. 16 s. $7\frac{43}{56}$ d.

Quest. 9. If 1 lb. of cochineal cost 1 l. 5 s. what will $36\frac{7}{10}$ lb. cost? *Ans.* 45 l. 17 s. 6 d.

Quest. 10. If 1 yard of broad-cloth cost $15\frac{5}{8}$ s. what will 4 pieces, each containing $27\frac{1}{7}$ yards, cost at that rate? *Ans.* 85 l. 14 s. $3\frac{3}{7}$ d.

Quest. 11. A mercer bought $3\frac{1}{2}$ pieces of silk, each piece containing $24\frac{2}{3}$ ells at 6 s. $\frac{3}{4}$ d. *per* ell; I demand the value of $3\frac{1}{2}$ pieces at that rate? *Ans.* 26 l. 3 s. $4\frac{3}{4}$ d.

In resolving the four next questions, observe the 8th rule of chapter 19.

Quest. 12. If $\frac{2}{3}$ of an ounce of silver cost 2 s. I demand the price of $11\frac{2}{3}$ lb. at that rate? *Ans.* 35 l.

Quest 13. If $1\frac{5}{7}$ lb. of gold is worth $61\frac{5}{7}$ l. Sterling, what is a grain worth at that rate? *Ans.* $1\frac{1}{2}$ d.

Quest. 14. If $\frac{3}{4}$ yard of silk is worth $\frac{3}{4}$ of $\frac{5}{6}$ l. what is the price of $15\frac{2}{5}$ ells Flemish? *Ans.* 9 l. 12 s. 6 d.

Quest.

Quest. 15. If $\frac{2}{3}$ of $\frac{3}{4}$ of a pound of cloves cost 6 s. $2\frac{2}{7}$ d. wh cost the C. weight at that rate? *Ans.* 69 l. 6 s. 8 d.

Note, That when the answers to the questions in this and the next chapter are given in fractions, they are given in their lowest terms.

C H A P. XXV.

The Rule of Three inverse in Fractions.

1. IT hath been already taught (in the 3d rule of the 11th chapter) how to discover when the fourth proportional number (to the three given numbers) is to be found out by the rule of three direct, and when by the rule of three inverse; to which rule the learner is now referred.

2. When in fractions you find a question to be solved by the rule of three inverse, *viz.* when the third term is the divisor; then having reduced the terms exactly, (according to the rules in chapter 24.), multiply the numerator of the third fraction into the denominators of the second and first fractions, and the product is a new denominator; then multiply the denominator of the third fraction into the numerators of the second and first fractions, and the product is a new numerator; which new fraction thus found is the answer to the question.

Quest. 1. If $\frac{3}{4}$ of a yard of cloth that is 2 yards wide will make a garment, how much of any other drapery that is $\frac{3}{5}$ of a yard wide will make the same garment? *Ans.* $2\frac{1}{2}$ yards.

Quest. 2. I lent my friend 46 l. for $\frac{4}{5}$ of a year, how much ought he to lend me for $\frac{7}{12}$ of a year? *Ans.* $63\frac{3}{35}$ l.

Quest. 3. If $\frac{2}{3}$ of a yard of cloth that is $2\frac{1}{3}$ yards wide will make any garment, what breadth is that cloth when $1\frac{3}{4}$ yards will make the same garment? *Ans.* $\frac{56}{63}$ of a yard wide.

Quest. 4. How many inches in length of a board that is 9 inches broad will make a foot square? *Ans.* 16 inches in length.

Quest. 5. If when the bushel of wheat cost $4\frac{3}{4}$ s. the

penny-loaf weighed $10\frac{2}{3}$ ounces, what will it weigh when the bushel cost $8\frac{9}{10}$ s.? *Ans.* $5\frac{185}{267}$ ounces.

Quest. 6. If 12 men can mow $24\frac{1}{2}$ acres in $10\frac{2}{3}$ days, in how many days will 6 men take do the same? *Ans.* in $21\frac{1}{3}$ days.

CHAP. XXVI.

Rules of Practice.

1. IN the single rule of three, when the first of the three numbers in the question (after they are disposed according to the 6th rule of chapter 10.) happeneth to be an unit, or 1, that question many times may be resolved far more speedily than by the rule of three; which kind of operation is commonly called *Practice*. And indeed it is of excellent use among merchants, tradesmen, and others, by reason of its speediness in finding a resolution to such kind of questions.

2. The chief questions resolvable by these brief rules may be comprehended under the seven general heads or cases following, *viz.*

When the given price of the integer consists,

1. Of farthings under 4.
2. Of pence under 12.
3. Of pence and farthings.
4. Of shillings under 20.
5. Of shillings, pence, and farthings.
6. Of pounds.
7. Of pounds, shillings, pence, and farthings.

It would be very convenient for the practical arithmetician to have by heart the several products of the 9 digits multiplied by 12; for his speedy reducing pence into shillings, and shillings into pence, which he may gain by the table on the margin.

12 times	1	is	12
	2		24
	3		36
	4		48
	5		60
	6		72
	7		84
	8		96
	9		108

3 Shillings

3. Shillings are practically reduced into pounds thus, *viz.* Cut off the figure standing in the place of units with a dash of the pen, and note it for shillings; then draw a line under the given number, and take half of the remaining figures after the first is cut off, and set them under the line, and they are so many pounds: but if the last figure is odd, then take the lesser half, and add 10 to the figure so cut off (as before) for shillings. As, if I were to reduce 43658 shillings into pounds; first I cut off the last figure (8) for shillings, then I take half of the remaining figures (4365) thus. Half of 4 is 2, which I put under the line; then half of three is 1; and because 3 is an odd number, I make the next figure 6 to be 16; and I go on, saying, Half of 16 is 8, and the half of 5 is 2, which is the last figure; wherefore because 5 is an odd number, I add 10 to the 8 I cut off, and it makes 18 s. So that I find it to be 2182 l. 18 s. as *per* margin.

4365	8
l.	*s.*
2182	18

4. It is likewise convenient that the learner be acquainted with the practical tables following; the first containing the aliquot or even parts of a shilling, the second containing the aliquot parts of a pound.

The even parts of a shilling.

d.		*s.*
6	is	$\frac{1}{2}$
4	is	$\frac{1}{3}$
3	is	$\frac{1}{4}$
2	is	$\frac{1}{6}$
$1\frac{1}{2}$	is	$\frac{1}{8}$
1	is	$\frac{1}{12}$

The even parts of a pound.

s.	*d.*		*l.*
10	0	is	$\frac{1}{2}$
6	8	is	$\frac{1}{3}$
5	0	is	$\frac{1}{4}$
4	0	is	$\frac{1}{5}$
3	4	is	$\frac{1}{6}$
2	6	is	$\frac{1}{8}$
2	0	is	$\frac{1}{10}$
1	8	is	$\frac{1}{12}$
1	0	is	$\frac{1}{20}$

5. *Case* 1. When the price of an integer is a farthing, then take the sixth part of the given number, which will be so many three-halfpences; and if any thing remains, is is farthings by the 8th rule of chapter 8. Then consider that three-halfpences is $\frac{1}{8}$ of a shilling, wherefore take the 8th part of them for shillings, and if any thing remain, they are so many three-halfpences: which reduce into pounds by

by the 3d rule foregoing. *Example*, What comes 67486 lb. to, at a farthing *per* lb.? First, I take $\frac{1}{6}$ of 67486, and it is 11247 three-halfpences, and 4 farthings, or 1 penny. Then $\frac{1}{8}$ of 11247 is 1405 s. and 7 remains, which is 7 three-halfpences, or 10½ d.; which, with the 4 farthings before, make 11½ d.; and 1405 shillings, by the third rule, is 70 l. 5 s.: in all 70 l. 5 s. 11½ d. for the answer. See the work.

	lb. *d.*
$\frac{1}{6}$	67486 at $\frac{1}{4}$ *per* lb.
$\frac{1}{8}$	11247—1
$\frac{1}{20}$	140|5—11½
	l. *s.* *d.*
	70—5—11½ *fac.*

Other examples follow.

$\frac{1}{6}$	8576 lb. at 1 qr.	$\frac{1}{6}$	6380 lb. at 1 qr.
$\frac{1}{8}$	1429——2 qrs.	$\frac{1}{8}$	1063——2 qrs.
$\frac{1}{20}$	17|8——8 d.	$\frac{1}{20}$	13|2——11 d.
	l. *s.* *d.*		*l.* *s.* *d.*
	8—18—8 *facit.*		6—12—11

6. When the price of the integer is 2 farthings, then take the third part of the given number for so many three-halfpences, and the remainder (if any) is halfpence; then take the eighth part of that for shillings, as before, *&c.*

Examples.

$\frac{1}{3}$	7368 lb. at 2 qrs.	$\frac{1}{3}$	8347 lb. at 2 qrs.
$\frac{1}{8}$	2456	$\frac{1}{8}$	2782——2 qrs.
$\frac{1}{20}$	30|7	$\frac{1}{20}$	37|7——9½ d.
	l. *s.*		*l.* *s.* *d.*
	15—7 *facit*		17 7 9½ *facit.*

7. When the price of the integer is 3 farthings, then take half the given number for three-halfpence, and if any thing remain, it is three farthings; then take the eighth of that for shillings, as before, *&c.*

½

$\frac{1}{2}$	4736 lb. at 3 qrs.
$\frac{1}{8}$	2368
$\frac{1}{20}$	29|6
	l. *s.*
	14—16 *facit.*

$\frac{1}{2}$	5425 lb. at 3 qrs.
$\frac{1}{8}$	2712——3 qrs.
$\frac{1}{20}$	33|9
	l. *s.* *d.* *qrs.*
	16—19—0—3 *facit.*

8. *Case* 2. When the given price of the integer is a part or parts of a ſhilling, (*viz.* pence), divide the given number of integers whoſe value is ſought, by the denomiinator of the fraction repreſenting the even part, and the quote is ſhillings; (always minding the 8th rule of the 8th chapter, for the value of the remainder); and thoſe ſhillings may be reduced into pounds by the 3d rule of this chapter. For example: Let it be required to find the value of 438 lb. at 3 d. *per* lb. I conſider that 3 d. is $\frac{1}{4}$ of a ſhilling, and 438 lb. will coſt ſo many three pences; wherefore I divide 438 by 4, the denominator of $\frac{1}{4}$, and the quote is 109 ſhillings, and 2 remains, which is 2 three pences, or 6 d.: the whole value is 5 l. 9 s. 6 d. as by the work appeareth.

$\frac{1}{4}$	438 lb. at 3 d.
$\frac{1}{20}$	10|9——6
	l. *s.* *d.*
	Facit 5—9—6

More examples follow.

	lb. *d.*
$\frac{1}{2}$	3574 at 6 *per* lb.
$\frac{1}{20}$	178|7
	89 l. 7 s. *facit.*

	lb. *d.*
$\frac{1}{6}$	5316 at 2 *per* lb.
$\frac{1}{20}$	88|6
	44 l. 6 s. *facit.*

	lb. *d.*
$\frac{1}{3}$	438 at 4 *per* lb.
$\frac{1}{20}$	14|6
	7 l. 6 s. *facit.*

	lb. *d.*
$\frac{1}{8}$	6389 at $1\frac{1}{2}$ *per* lb.
$\frac{1}{20}$	79|8—$7\frac{1}{2}$ d.
	39 l. 18 s. $7\frac{1}{2}$ d. *fac.*

	lb. *d.*
$\frac{1}{4}$	879 at 3 *per* lb.
$\frac{1}{20}$	21|9—9
	10 l. 19 s. 9 d. *facit.*

	lb. *d.*
$\frac{1}{12}$	818 at 1 *per* lb.
$\frac{1}{20}$	6|8——2 d.
	3 l. 8 s. 2 d. *facit.*

9. If the price of the integer be pence under 12, and yet not an even part, then it may be divided into even parts; and so the parts of the given number taken accordingly, and added together, as if it were 5 d. which is 3 d. and 2 d. *viz.* $\frac{1}{4}$ and $\frac{1}{6}$ of a shilling; first take $\frac{1}{4}$ of the given number, and then $\frac{1}{6}$ thereof, and add them together, and their sum is the answer in shillings; still observing rule 8. of chap. 8. for the remainder, (if any be); then bring the shillings into pounds by the 3d rule foregoing. Likewise 7 d. is $\frac{1}{3}$ and $\frac{1}{4}$, so 9 d. is $\frac{1}{2}$ and $\frac{1}{4}$, and 10 d. is $\frac{1}{2}$ and $\frac{1}{3}$, and 11 d. is $\frac{1}{3}$ and $\frac{1}{3}$ and $\frac{1}{4}$ of a shilling. Or else, many times your work may be shortened thus, *viz.* When the said given price is to be divided into even parts of a shilling or of a pound; after you have taken the first even part, the other may be an even part of that part; as in the next example, where is given 439 lb. at 5 d. *per* lb. Now, I may divide it thus, *viz.* into 4 d. 1 d.; and 4 d. being $\frac{1}{3}$ of a shilling, and 1 d. being $\frac{1}{4}$ of 4 d. I first take $\frac{1}{3}$ of 439 lb. and it gives 146 s. 4 d.; and for the 1 d. I take $\frac{1}{4}$ of 146 s. 4 d. which is 36 s. 7 d.; which in all comes to 9 l. 2 s. 11 d. Examples follow.

	lb. *d.*
$\frac{1}{3}$	439 at 5 *per* lb.
$\frac{1}{4}$	146—4
	36—7
$\frac{1}{20}$	18|2—11
	9 l. 2 s. 11 d. *facit.*

	Yards. *d.*
$\frac{1}{2}$	417 at 9 *per* yard.
$\frac{1}{2}$	208—6
	104—3
$\frac{1}{20}$	31|2—9
	15 l. 12 s. 9 d. *facit.*

	Ells. *d.*
	587 at 7 *per* ell.
$\frac{1}{3}$	195—8
$\frac{1}{4}$	146—9
$\frac{1}{20}$	34|2—5
	17 l. 2 s. 5 d. *facit.*

	Ells. *d.*
	386 at 10
$\frac{1}{2}$	193
$\frac{1}{3}$	128—8
$\frac{1}{20}$	32|1—8
	16 l. 1 s. 8 d. *facit.*

Yards.

	Yards. d.		*lb.* d.
	836 at 8 *per* yard.		534 at 11
$\frac{1}{3}$	278—8	$\frac{1}{3}$	178
$\frac{1}{3}$	278—8	$\frac{1}{3}$	178
		$\frac{1}{4}$	133—6d.
$\frac{1}{20}$	55|7—4	$\frac{1}{20}$	48|9—6
	27 l. 17 s. 4 d. *facit.*		24 l. 9 s. 6 d. *facit.*

10. *Case* 3. When the price of the integer is pence and farthings, if it make an even part of a shilling, work as before: but if they are uneven, as penny-farthing, penny three-farthings, 2 d. 1 qr. or 2 d. 3 qrs. 3 d. 3 qrs. or the like; then first work for some even part, and then consider what part the rest is of that even part, and divide that quotient thereby; then add them together, and reduce them to pounds, as before.

Example. 3470 lb. at 1 d. 1 qr. *per* lb. First, I work for the penny, by dividing 3470 lb. by 12, for 1 d. is $\frac{1}{12}$ of a shilling, and the quote is 289 s. 2 d. Then I conceive that one farthing is the $\frac{1}{4}$ of a penny; and the value of 1 farthing will be $\frac{1}{4}$ of the value of a penny; and therefore I take $\frac{1}{4}$ of 289 s. 2 d. which is 72 s. 3 d. 2 qrs. and add them together, and they are 18 l. 1 s. 5 d. 2 qrs. as by the margin. Other examples of the same nature follow.

	lb. d. qr.
$\frac{1}{12}$	3470 at 1—1
$\frac{1}{4}$	289—2
	72—3—$\frac{1}{2}$
$\frac{1}{20}$	36|1—5$\frac{1}{2}$
	l. *s.* *d.* *qrs.*
	18—1—5—2

	lb. d.		*Yards.* d.
$\frac{1}{12}$	4360 at 1$\frac{1}{4}$	$\frac{1}{12}$	859 at 1$\frac{1}{8}$
$\frac{1}{4}$	363—4	$\frac{1}{8}$	71—7 qr.
	90—10		8—11—1$\frac{1}{2}$
$\frac{1}{20}$	45|4—2	$\frac{1}{20}$	8|0—6—1$\frac{1}{2}$
	l. *s.* *d.*		*l.* *s.* *d.* *qr.*
	22—14—2 *facit.*		4—0—6—1$\frac{1}{2}$ *facit.*

lb.

	lb. *d.*
$\frac{1}{6}$	485 at $2\frac{1}{4}$
8	80—10
	10—$1\frac{1}{4}$
$\frac{1}{20}$	9|0—$11\frac{1}{4}$
	4 l. 10s. $11\frac{1}{4}$d. *facit.*

	Yards. *d.*
$\frac{1}{2}$	520 at $7\frac{1}{2}$
4	260
	65
$\frac{1}{20}$	32|5
	16 l. 5 s. *facit.*

$\frac{1}{6}$	654 lb. at $2\frac{1}{4}$ d.
$\frac{1}{4}$	109
	27—3
$\frac{1}{20}$	13|6—3
	6 l. 16 s. 3 d. *facit.*

$\frac{1}{2}$	137 yds at $10\frac{1}{2}$ d.
$\frac{1}{2}$	68—6
$\frac{1}{2}$	34—3
	17—$1\frac{1}{2}$
$\frac{1}{20}$	11|9—$10\frac{1}{2}$
	5 l. 19 s. $10\frac{1}{2}$d. *fac.*

11. *Case* 4. When the price of the integer is 2 s. then cut off the figure in the place of units of the given number, and double it for shillings, and the figures on the other hand are pounds. *Example:* 436 yards at 2 s. *per* yard; cut off the last figure 6, and double it, it makes 12 shillings; and the other two figures, *viz.* 43, are so many pounds: so that their value is 43 l. 12 s. as *per* margin.

43|6
43 l. 12 s.

12. Hence it is evident, that when the given price of an integer is an even number of shillings, then if you take half of that (even) number of shillings, and multiply the given number of integers thereby, doubling the first figure of the product, and setting it apart for shillings, the rest of the product will be pounds; which pounds and shillings are the value sought. *Example.* What cost 536 yards at 8 s. *per* yard? To resolve which, I take $\frac{1}{2}$ of 8 s. (the price of a yard), which is 4, and multiply 536 thereby; saying, 4 times 6 is 24; then I double the first figure 4, which makes 8, for shillings, and carry 2 to the next product, *&c.* and I find the rest of the product to be 214, which I note for pounds: so the value of 536 yards at 8 s. *per* yard, is 214 l. 8 s. as *per* margin. More examples follow.

536 yds at 8 s.
214 l. 8 s.

56 yds at 6 s. *per* yd.	420 yds at 12 s. *per* yd.
16 l. 16 s. *facit.*	252 l. *facit.*
123 yds at 4 s. *per* yd.	326 yds at 14 s. *per* yd.
24 l. 12 s. *facit.*	228 l. 4 s. *facit.*
48 ells at 8 s. *per* ell.	48 yds at 16 s. *per* yd.
19 l. 4 s. *facit.*	38 l. 8 s. *facit.*
84 yds at 10 s. *per* yd.	52 yds at 18 s. *per* yd.
42 l. *facit.*	46 l. 16 s. *facit.*

13. If the given price of the integer is an odd number of shillings, then work first for the even number of shillings, by the last rule; and for the odd shilling take $\frac{1}{20}$ of the given number of integers, according to the 3d rule of this chapter, and add them together, and you have your desire. Examples follow.

Yds. *s.*	*Ells.* *s.*
422 at 3 *per* yd.	431 at 13
l. *s.*	*l.* *s.*
42——4	258——12
21——2	21——11
63——6 *facit.*	280—— 3 *facit.*
Ells. *s.*	*Ells.* *s.*
516 at 7 *per* ell.	324 at 17 *per* ell.
l. *s.*	*l.* *s.*
154——16	259——4
25——16	16——4
180——12 *facit.*	275——8 *facit.*

14. Except when the given price of the integer is 5 s.; for then it is sooner answered by taking $\frac{1}{4}$ of the given number whose value is sought, as in the following examples.

	Yds. *s.*		*Ells.* *s.*
$\frac{1}{4}$	436 at 5 *per* yd	$\frac{1}{4}$	206 at 5 *per* ell.
	109 l. *facit.*		51 l. 10 s. *facit.*

15. *Case* 5. When the given price of an integer is ſhillings and pence, or ſhillings, pence, and farthings; then if the ſhillings and pence be an even part of a pound, divide the given number of integers whoſe value you ſeek, by the denominator of that fraction repreſenting that even part. As for example: What is the price of 384 yards at 6 s. 8 d. *per* yard? Here I conſider that 6 s. 8 d. is $\frac{1}{3}$ of a pound; wherefore I divide 384 by 3, and the quote is the anſwer, *viz.* 128 l.; ſo that 384 yards at 6 s. 8 d. *per* yard, amount to 128 l. as *per* margin, ſtill obſerving the 8th rule of the 8th chapter.

	Yds. *s.* *d.*
$\frac{1}{3}$	384 at 6—8 *per* yd.
	128 l. *facit.*

More Examples follow.

$\frac{1}{3}$	438 ells at 6 s. 8 d.	$\frac{1}{8}$	443 yds at 2 s. 6 d.
	146 l. *facit.*		55 l. 7 s. 6 d. *facit.*
$\frac{1}{6}$	1125 ells at 3 s. 4 d.	$\frac{1}{12}$	726 yds at 1 s. 8 d.
	187 l. 10 s. *facit.*		60 l. 10 s. *facit.*

16. When the given value of the integer is ſhillings and pence, &c. and not an even part of a pound, yet many times it may be divided into even parts. Thus 6 s. 6 d. is 4 s. and 2 s. 6 d. For the 4 s. work according to the 12th rule foregoing, and for the 2 s. 6 d. take the eighth part of the given number, and add them together, then their ſum is the value required.

So 8 s. 6 d. will be divided into 6 s. and 2 s. 6 d. and the price of the given number may be found out as before, &c. Examples follow.

	Yds. *s.* *d.*		*Ells* *s.* *d.*
	386 at 8—8		540 at 5—4
$\frac{1}{3}$	128 l.—13—4	2 s.	54——0 s.
2 s.	38 —12—0	$\frac{1}{6}$	90——0 s.
	167 l. 5 s. 4 d. *facit.*		144 l. *facit.*

Ells.

	Ells s. d.			Yds. s. d.
	427 at 8—6			386 at 14—8
6 s.	128 l.—2—0		8 s.	154 l.— 8—0
$\frac{1}{8}$	53 —7—6		$\frac{1}{3}$	128 —13—4
	181 l. 9 s. 6 d. *facit.*			283 l.— 1—4 *fac.*

17. When the given price of an integer is shillings and pence, and you cannot readily divide them according to the last rule, then multiply the given number whose value you seek, by the number of shillings in the price of the integer; and then for the pence work by the 8th rule foregoing; then add the numbers together, and their sum is the value sought in shillings. As for example: What is the value of 392 yards at 6 s. 9 d. *per* yard? Here 6 s. 9 d. cannot be made an even part; nor indeed can it be divided into even parts of a pound; wherefore I multiply the given number of yards, 392 by 6, for the 6 s. the product is 2352 shillings; then for the 9 d. I divide it into 6 d. and 3 d. and work for them by the 8th rule foregoing, and at last add the shillings together, they make 2646 s.; and by the 3d rule they are reduced to 132 l. 6 s. the value of 392 yards at 6 s. 9 d. *per* yard. See the work in the margin.

	Yds. s. d.
	392 at 6—9
6 s	2352
$\frac{1}{2}$	196
$\frac{1}{4}$	98
$\frac{1}{20}$	264|6
	132 l. 6 s. *facit.*

Other Examples follow.

	lb. s. d.			Ells. s. d.
	480 at 4—10			732 at 12—7
				12
4 s	1920		12 s	8784
$\frac{1}{2}$	240		$\frac{1}{3}$	244
$\frac{1}{3}$	160		$\frac{1}{4}$	183
$\frac{1}{20}$	232|0		$\frac{1}{20}$	921|1
	116 l. *facit.*			460 l. 11 s. *facit.*

18. When the given price of the integer is shillings, pence, and farthings, then multiply the given number of integers by the number of shillings contained in the value of the integer; and for the pence and farthings follow the 10th rule of this chapter.

Examples.

	Yds. *s.* *d.*
	438 at 8—6¾
8 s	3504
½	219
⅛	27—4½ *d.*
$\frac{1}{20}$	375|0—4½
	187 l. 10s. 4½d. *fac.*

	Ells. *s.* *d.*
	270 at 14—2¾
	148
	37
14 s	5180 *d.*
$\frac{1}{6}$	61—8
¼	15—5
½	7—8½
$\frac{1}{20}$	526|4—9½
	263 l. 4s. 9½d. *facit.*

	Ells. *s.* *d.*
	136 at 9—2½
9 s	1224—0 d.
$\frac{1}{6}$	22—8
¼	5—8
$\frac{1}{20}$	125|2—4
	62 l. 12 s. 4d. *facit.*

	Ells. *s.* *d.*
	431 at 2—4½
2 s	862
¼	107—9 d.
½	53—10½
$\frac{1}{20}$	102|3—7½
	51 l. 3 s. 7½ d. *facit.*

19. *Case* 6. When the given value of the integer is pounds, then multiply the number of integers whose value is sought, by the price of the integer, and the product is the answer in pounds.

Examples.

C. *l.*	*C.* *l.*
42 at 2 *per* C.	13 at 8 *per* C.
84 l. *facit.*	104 l. *facit.*

C. *l.*	*C.* *l.*
30 at 3 *per* C.	48 at 12 *per* C.
90 l. *facit.*	576 l. *facit.*

20.

20. *Case* 7. If the price of the integer is pounds and ſhillings, then for the pounds work as in the laſt rule, and for the ſhillings as in the 12th and 13th rules foregoing; then add the numbers produced from them both, and the ſum is the value ſought.

Examples.

	C. *l.* *s.*
	46 at 2—4
2 l.	92 *s.*
4 s.	9—4
	101 l. 4 s. *facit.*

	Groſs. *l.* *s.*
	82 at 4—10
4 l.	328
10 s.	41
	369 l. *facit.*

	Groſs. *l.* *s.*
	58 at 3—7
3 l.	174 *s.*
6 s.	17—8
1 s.	2—18
	194 l. 6 s. *facit.*

	Groſs. *l.* *s.*
	26 at 3—15
3 l.	78
14 s.	18—4
	1—6
	97 l. 10 s. *facit.*

21. When the given price of an integer conſiſts of pounds, ſhillings, pence, and farthings, then firſt work for the ſhillings, pence, and farthings, according to the 18th rule of this chapter; and find the total value of the given number, as if there were no pounds; then work with the pounds according to the 19th rule of this chapter; and add the numbers thus found, and their ſum is the total value required.

Examples of this rule follow.

	C. *l.* *s.* *d.*
	213 at 1—13—4½
	639
	213
13 s.	2769—*d.*
3 d.	53—3
1½ d.	26—7½
	284|8—10½
	142 l. 8 s. 10½ d.
1 l.	213
	355 l. 8 s. 10½ d. *fac.*

C. *l.* *s.* *d.*	
37 at 3—8—10½	
296—*d.*	8 s.
18—6	6 d.
9—3	3 d.
4—7½	1½ d.
32|8—4½	
16 l. 8 s. 4½ d.	
111	3 l.
127 l. 8 s. 4½ d. *facit.*	

	Gross.	*l.*	*s.*	*d.*
	416 at	2 —	9 —	$3\frac{3}{4}$
9 s.	3744			
3 d.	104			
$\frac{3}{4}$ d.	26			
	387|4			
	193 l. 14 s.			
2 l.	832			
	1025 l. 14 s. *facit.*			

Gross.	*l.*	*s.*	*d.*	
48 at	3 —	15 —	$11\frac{1}{2}$	
240				
48				
720				15 s.
24				6 d.
16				4 d.
6				$1\frac{1}{2}$ d.
76|6—				
38—				
144				3 l.
182 l. 6 s. *facit.*				

22. When there is given the value of an integer, and it is required to know the value of many such integers, together with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an integer; then first (by the former rules) find out the value of the given number of integers; and then for $\frac{1}{4}$ of an integer take $\frac{1}{4}$ of the given value of the integer; or for $\frac{1}{2}$ take $\frac{1}{2}$ of the given value of the integer; and for $\frac{3}{4}$, first take $\frac{1}{2}$ of the given value, and then $\frac{1}{2}$ of that $\frac{1}{2}$, setting each part under the precedent: then adding them together, their sum will be the required value of the integers and their parts. *Example*: What is the value of $116\frac{1}{2}$ yards at 4s. 6d. *per* yard? To give an answer, first I work for the value of 116 yards, by the 15th rule foregoing; and then for the $\frac{1}{2}$ yard I take $\frac{1}{2}$ of 4s. 6d. which is 2s. 3d. and add to the rest found as before, then is that sum the total value of $116\frac{1}{2}$ yards at 4s. 6d. *per* yard; which I find to amount to 26 l. 4 s. 3 d. as by the work in the margin.

Yds.	*s.*	*d.*	
$116\frac{1}{2}$ at	4 —	6	
11 l.	12 s.		2 s.
14 l.	10 s.		2 s. 6d.
	2s. 3d.		$\frac{1}{2}$ yd.
26 l. 4 s. 3 d. *facit.*			

Other

Other examples follow.

324¼ yards at 4 s. 10 d.

1296	4 s.
162	6 d.
108	4 d.
1——2½ d.	¼ yd.

156|7 s. 2½ d.

78 l. 7 s. 2½ d. *facit.*

720½ yards at 6 s. 8 d.

240 l. 3 s. 4 d. *facit.*

228¾ ells at 12 s. 11 d.

2736	12 s.
76	4 d.
76	4 d.
57	3 d.
6——5½ d.	½ ell.
3——2¾ d.	¼ ell.

295|4 s. 8¼ d.

147 l. 14 s. 8¼ d. *facit.*

C. qrs. lb. l. s. C.
28—3—14 at 1 10 *per*

28 l.	1 l.
14	10 s.
0—15 s.	½ C.
7 6 d.	¼ C.
3 9	14 lb.

43 l. 6 s. 3 d. *facit.*

Many more queſtions might be ſtated, and ſeveral other rules of practice might be ſhewn, according to the method of divers authors; but what have been delivered here, are ſufficient for the practical arithmetician in all caſes whatſoever.

CHAP. XXVII.

The Rule of Barter.

1. BArter is a rule among merchants, which (in the exchanging of one commodity for another) informs them ſo to proportion their rates, as that neither may ſuſtain loſs.

2. To reſolve queſtions in barter, it will not be difficult to him that is acquainted with the golden rule, or rule of three, it being altogether uſed in reſolving ſuch queſtions.

Queſt. 1. Two merchants (*viz.* A and B) barter. A hath 13 C. 3 qrs. 14 lb. of pepper, at 2 l. 16 s. *per* C. and B hath cotton at 9 d. *per* lb. I demand how much cotton

cotton B must give A for his pepper? *Ans.* 9 C. 1 qr.

First find by the rule of three, or the rules of practice, foregoing, how much the pepper is worth; saying,

If 1 C. cost 2 l. 16 s. what will 13 C. 3 qrs. 14 lb. cost? *Ans.* 38 l. 17 s.

Secondly, By the rule of three, say, If 9 d. buy 1 lb. of cotton, how much will 38 l. 17 s. buy? *Ans.* $9\frac{1}{4}$ C.; and so much cotton must B give to A for 13 C. 3 qrs. 14 lb. of pepper, at 2 l. 16 s. *per* C. when the cotton is worth 9 d. *per* lb.

Quest. 2. Two merchants (A and B) barter. A hath ginger worth 1 lb. 17 s. 4 d. *per* C. but in barter he will have 2 l. 16 s. *per* C. B hath nutmegs worth 5 l. 12 s. *per* C.; now I demand how B must rate his nutmegs *per* C. to make his gain in barter equal to that of A? *Ans.* 8 l. 8 s.

Say, by the rule of three, If 1 l. 17 s. 4 d. require 2 l. 16 s. in the barter, what will 5 l. 12 s. require in barter? *Facit* 8 l. 8 s.

Quest. 3. A and B barter. A hath 120 yards of broad-cloth, worth 6 s. *per* yard, but in barter he will have 8 s. *per* yard. B hath shalloon worth 4 s. *per* yard: now I demand how many yards of shalloon B must give A for his broad cloth, making his gain in barter equal to that of A? *Ans.* 180 yards of shalloon.

First (as in the last question) find out how B ought to sell his shalloon in barter, *viz.* say, If 6 s. require 8 s. what will 4 s. require? *Ans.* 5 s. 4 d.

Thus you see that B must sell his shalloon in barter at 5 s. 4 d. if A sell his broad-cloth at 8 s. *per* yard.

It remaineth now to find out how much shalloon B must give for 120 yards of broad-cloth; which, after the same method used to resolve the first question of this chapter, is found to be 180; and so many yards of shalloon must B give A for the 120 yards of broad-cloth.

Quest. 4. A and B bartered. A had 14 C. of sugar worth 6 d. *per* lb. for which B gave him 1 C. 3 qrs. of cinnamon; I demand how B rated his cinnamon *per* lb.? *Ans.* 4 s. *per* lb.

Quest. 5. A and B barter. A hath 4 tuns of brandy, worth 37 l. 16 s. *per* tun in ready money; but in barter he

he hath 56 l. 8 s. *per* tun; and A giveth B 21 C. 2 qrs. 11 1/7 lb. ginger for the 4 tuns of brandy; I desire to know how much B sold his ginger in barter *per* C. and how much it was worth in ready money? *Ans.* For 9 l. 6 s. 8 d. in barter, and it is worth 7 l. *per* C. in ready money.

Quest. 6. A and B barter. A hath 320 dozen of candles, at 4 s. 6 d. *per* dozen; for which B giveth him 30 l. in money, and the rest in cotton at 8 d. *per* lb. I demand how much cotton he must give him more than the 30 l.? *Ans.* 11 C. 1 qr.

Quest. 7. A and B barter. A hath 608 yards of broad-cloth, worth 14 s. *per* yard, for which B giveth him 125 l. 12 s. ready money, and 85 C. 2 qrs. 24 lb. of bees-wax; now I desire to know how he reckoned his wax *per* C.? *Ans.* 3 l. 10 s. *per* C.

CHAP. XXVIII.

Questions in Loss and Gain.

Quest. 1. A Merchant bought 436 yards of broad-cloth for 8 s. 6 d. *per* yard, and selleth it again at 10 s. 4 d. *per* yard; now I desire to know how much he gained in the sale of the 436 yards? *Ans.* 39 l. 19 s. 4 d.

First, find out by the rule of three, or by practice, how much the cloth cost him at 8 s. 6 d. *per* yard, which I find to be 185 l. 6 s.; then by the same rule find out how much he sold it for, *viz.* 225 l. 5 s. 4 d.; then subtract 185 l. 6 s. which it cost him, from 225 l. 5 s. 4 d. which he sold it for, and there remaineth 39 l. 19 s. 4 d. for his gain in the sale thereof.

Otherwise it may sooner be resolved thus: First, find out how much he gained *per* yard, *viz.* subtract 8 s. 6 d. which he gave *per* yard, from 10 s. 4 d. which he sold it for *per* yard, the remainder is 1 s. 10 d. for his gain *per* yard. Then say,

If 1 yard gain 1 s. 10 d. what will 436 yards gain? The answer by practice, or the rule of three, is 39 l. 19 s. 4 d. as was found before.

Quest. 2. A draper bought 124 yards of holland cloth, for

for which he gave 31 l. I desire to know how he must sell it *per* yard, to gain 10 l. 6 s. 8 d. in the whole sale of the 124 yards? *Ans.* At 6 s. 8 d. *per* yard.

Add the price which it cost him (*viz.* 31 l.) to his intended gain, (*viz.* 10 l. 6 s. 8 d.), the sum is 41 l. 6 s. 8 d. Then say,

If 124 yards require 41 l. 6 s. 8 d. what will 1 yard require? By the rule of three, I find the answer 6 s. 8 d.

Quest. 3. A grocer bought 3 C. 1 qr. 14 lb. of cloves, which cost him 2 s. 4 d. *per* lb. and sold them for 52 l. 14 s. I desire to know how much he gained in the whole? *Ans.* 8 l. 12 s.

Quest. 4. A draper bought 86 kerseys for 129 l. I demand how he must sell them *per* piece to gain 15 l. in laying out 100 l. at that rate? *Ans.* 1 l. 14 s. 6 d. *per* piece. For

As 100 is to 115 l. so is 129 l. to 148 l. 7 s.

So that, by the proportion above, I have found how much he must receive for the 86 kerseys to gain after the rate of 15 l. *per* C. Then to find how he must sell them *per* piece, I say,

As 86 pieces are to 148 l. 7 s. so is 1 piece to 1 l. 14 s. 6 d. which is the number sought.

Quest. 5. A grocer bought $4\frac{1}{2}$ C. of pepper for 15 l. 17 s. 4 d. and (it proving to be damnified) he is willing to lose 12 l. 10 s. *per cent.* I demand how he must sell it *per* lb.? *Ans.* 7 d. *per* lb.

Subtract 12 l. 10 s. the loss of 100 l. from 100 l. and there remains 87 l. 10 s. Then say,

As 100 l. is to 87 l. 10 s. so is 15 l. 17 s. 4 d. to 13 l. 17 s. 8 d. so much as he must sell it all for, to lose after the rate propounded. Then to know how he must sell it *per* lb. I say,

As $4\frac{1}{2}$ C. is to 13 l. 17 s. 8 d. so is 1 lb. to 7 d.

Quest. 6. A plummer sold 10 fodder of lead (the fodder containing $19\frac{1}{2}$ C.) for 204 l. 15 s. and gained after the rate of 12 l. 10 s. *per* 100 l. I demand how much it cost him *per* C.? *Ans.* 18 s. 8 d.

To resolve this question, add 12 l. 10 s. (the gain *per cent.*) to 100 l. and it makes 112 l. 10 s.; then say,

As 112 l. 10 s. is to 100 l. so is 204 l. 15 s. to 182 l.

Which

Which 182 l. is the sum it cost him in all. Then reduce your 10 fodders to half-hundreds, and it makes 390. Then say,

As 390 half-hundreds is to 182 l. so is 2 half-hundreds to 18 s. 8 d. the price of 2 half-hundreds, or one C. weight, and so much it stood him *per* C. weight.

Quest. 7. A merchant bought 8 tuns of wine, which being sophisticated, he selleth for 400 l. and loseth after the rate of 12 l. in receiving 100 l.: now I demand how much it cost him *per* tun? and how he selleth it *per* gallon to lose after the said rate? *Ans.* It cost 56 l. *per* tun, and he must sell it at 3 s. 11 d. $2\frac{10}{21}$ qrs. *per* gallon to lose 12 l. in receiving 100 l.

To resolve this question, I consider, in the first place, that in receiving 100 l. he loseth 12 l. therefore 100 l. comes in for 112 l. laid out; wherefore to find out how much he laid out for the whole, I say,

As 100 l. is to 112 l. so is 400 l. to 448 l. and so much the 8 tuns cost him. Then, to find how much it cost *per* tun, I say,

As 8 is to 448 l. so is 1 to 56 l. the price it cost *per* tun.

Now, to find how he must sell it *per* gallon, reduce the 8 tuns into gallons, they make 2016: then say,

As 2016 gallons is to 400 l. so is 1 gallon to 3 s. 11 d. $2\frac{10}{21}$ qrs. the price he must sell it at *per* gallon to lose as aforesaid.

Quest. 8. A merchant bought 8 tuns of wine, which being sophisticated, he is willing to sell for 400 l. and loseth at that rate 12 l. in laying out 100 l. upon the same; now I demand how much it cost him *per* tun?

Here I consider, that for 100 l. laid out he receiveth but 88 l.; wherefore to find what the 8 tuns cost him, I say,

As 88 l. is to 100 l. so is 400 l. to $454\frac{6}{11}$ the price it all cost him. Then to find how much *per* tun, I say,

As 8 is to $454\frac{6}{11}$, so is 1 to $56\frac{9}{11}$, or 56 l. 16 s. 4 d. $2\frac{5}{11}$ qrs. *per* tun.

C H A P. XXIX.

Equation of Payments.

1. EQuation of payments is that rule amongst merchants, whereby we reduce the times for payments

ments of several sums of money to an equated time for payment of the whole debt, without damage to debtor or creditor. And the rule is,

2. Multiply the sums of each particular payment by its respective time; then add the several products together, and their sum divide by the total debt; and the quotient thence arising is the equated time for the payment of the whole debt. *Example:*

Quest. 1. A is indebted to B in the sum of 130 l. whereof 50 l. is to be paid at 2 months, and 50 l. at 4 months, and the rest at 6 months; now they agree to make one payment of the total sum; the question is, What is the equated time for payment, without damage to debtor or creditor?

To resolve this question, I multiply each payment by its time, *viz.*

50 l. multiplied by 2 months, produceth —	100
50 l. multiplied by 4 months, produceth —	200
30 l. multiplied by 6 months, produceth —	180
The sum of the products is — —	480

Then I divide 480 (the sum of the products by 130 (the total debt), and the quotient is $3\frac{9}{13}$ months, for the time of paying that whole debt.

Quest. 2. A merchant hath owing him 1000 l. to be paid as followeth, *viz.* 600 l. at 4 months, 200 l. at 6 months, and the rest (which is 200 l.) at 12 months, and he agreeth with his debtor to make one payment of the whole: I demand the time of payment without damage to debtor or creditor?

600 l. multiplied by 4 months, is — —	2400
200 l. multiplied by 6 months, is — —	1200
200 l. multiplied by 12 months, is — —	2400
The sum of the products is — — —	6000

And the sum of the products (6000) being divided by the whole debt (1000 l.), quotes 6 months for the time of payment of the whole debt.

The

The proof of the rule of equation of payments.

3. The truth of this rule is thus manifeſt, if the intereſt of that money which is paid (by the equated time) after it is due, be equal to the intereſt of that money, which (by the equated time) is paid ſo much ſooner than it is due at any rate *per* C. then the operation is true, otherwiſe not. For example:

In the laſt queſtion 600 l. ſhould have been paid at 4 months, but is not diſcharged till 6 months, (that is, 2 months after it is due); wherefore its intereſt for 2 months at 6 *per cent. per ann.* is 6 l.; and then 200 l. was to be paid at 6 months, which is the equated time for its payment, therefore no intereſt is reckoned for it: but 200 l. ſhould have been paid at 12 month, but is paid at 6 months, which is 6 months ſooner than it ought; wherefore the intereſt of 200 l. for 6 months, is 6 l. (accounting 6 l. *per cent. per ann.*), which is equal to the intereſt of 600 l. for 2 months: wherefore the work is right.

Queſt. 3. A merchant hath owing him a certain ſum to be diſcharged at three equal payments, *viz.* $\frac{1}{3}$ at two months, $\frac{1}{3}$ at four months, and $\frac{1}{3}$ at eight months; the queſtion is, What is the equated time for the payment of the whole debt?

In queſtions of this nature (*viz.* where the debt is divided into equal or unequal parts) each of the parts is to be multiplied by its time, and the ſum of the products is the anſwer.

$\frac{1}{3}$ multiplied by 2 months, produceth $\frac{2}{3}$
$\frac{1}{3}$ multiplied by 4 months, produceth $1\frac{1}{3}$
$\frac{1}{3}$ multiplied by 8 months, produceth $2\frac{2}{3}$

The ſum of the product is — $4\frac{2}{3}$

which is $4\frac{2}{3}$ months for the equated time of payment.

If inſtead of the fractions repreſenting the parts, you had wrought by the numbers themſelves (repreſented by thoſe parts), according to the firſt and ſecond example, it would have been the ſame anſwer; and ſuppoſe the debt had been 90 l. then $\frac{1}{3}$ of it is 30 l. for each payment, *viz.* at 2, 4, and 8 months. Then

30 l. multiplied by 2 months, produceth 60
30 l. multiplied by 4 months, produceth 120
30 l. multiplied by 8 months, produceth 240

The ſum of the products is — 420

which divided by 90 (the whole debt), quoteth $4\frac{60}{90}$, or $4\frac{2}{3}$ months, as before.

Queſt. 4. A merchant oweth a ſum of money to be paid $\frac{1}{2}$ at 5 months, and $\frac{1}{4}$ at 8 months, and $\frac{1}{4}$ at 10 months, and he agreeth with his creditor to make one total payment; I demand the time without damage to debtor or creditor? Work as in the laſt queſtion, and you will find the anſwer to be 7 months.

Queſt. 5. A is indebted to B 640 l. whereof he is to pay 40 l. preſent money, 350 l. at 3 months, and the reſt (*viz.* 250 l.) at 8 months, and they agree to make an equated time for the whole payment; now I demand the time?

In queſtions of this nature (*viz.* where there is ready money paid) you are in multiplying to neglect the money that is to be paid preſent, and work with the reſt, as is before directed; and divide the ſum of the products by the whole debt, and the quote is the anſwer: for here 40 l. is to be paid preſent, and hath no time allowed, and, according to the rule, it ſhould be multiplied by its time, which is (0); therefore 40 times 0 is 0, which neither augmenteth nor diminiſheth the dividend: wherefore to proceed (according to direction), I ſay,

350 by 3 months, produceth — — 1050
250 by 8 months, produceth — — 2000

The ſum of the products is —— 3050

which divided by 640, the whole debt, the quote is $4\frac{49}{64}$ months, the time of payment.

Queſt. 6. A is indebted to B in a certain ſum, half whereof is to be paid preſent money, one third at 6 months, and the reſt at 8 months; now I demand the equated time for payment of it all? *Anſ.* $3\frac{1}{3}$ months is the time of payment.

Queſt.

Quest. 7. A is indebted to B 120 l. whereof $\frac{1}{3}$ is to be paid at 3 months, $\frac{1}{4}$ at 6 months, and the rest at 9 months; what is the equated time for the payment of the whole sum? *Ans.* At $6\frac{1}{4}$ months.

Quest. 8. A is indebted to B 420 l. which is due at the end of 6 months, but A is willing to pay him 140 l. present, provided he can have the remainder forborn so much the longer to make satisfaction for his kindness; which is agreed upon: I desire to know what time ought to be allotted for the payment of the 280 l. remaining?

To resolve this question, first, find out what is the interest of 140 l. for the time it was paid before it was due at 6 *per cent.* or any other rate, (*viz.* 6 months), and you will find it to be 4 l. 4 s. Then it is evident, that the remaining 280 l. must be detained so much longer than 6 months, as the while it may eat out that interest, *viz.* 4 l. 4 s.; which is thus found out, *viz.* First, see what is the interest of 280 l. for a month, or any other time; but here we will take one month, and its interest for one month is 28 s.

Then, by the rule of three, say,

As 28 s. is to 1 month, so is 84 s. to 3 months; so that the 280 l. remaining must be kept 3 months beyond its first time of payment, (*viz.* 6 months); which added thereto, makes 9 months; at the end of which time A ought to make payment of the remainder.

CHAP. XXX.

Exchange.

1. THE rule of exchange informeth merchants how to exchange monies, weights, or measures of one country into (or for) the monies, weights, or measures of another country; and when the rate, reason, or proportion betwixt the money, weights, or measures of different countries is known, it will not be difficult for the practitioner, that is well acquainted with the rule of proportion, (or rule of three), to resolve any question, wherein it is required to exchange a given quantity of the one kind into the same value of another kind.

2. In questions of exchange there is always a comparison made between the coins, *&c.* of two countries (or kinds) or more.

3. In questions where there is a comparison made between two things (whether they be monies, weights, *&c.*) of different kinds (or countries), there may be a solution found by a single rule of three; as may appear by the following example.

Quest. 1. A merchant at London delivered 370 l. Sterling to receive the same at Paris in French crowns, the exchange $3\frac{1}{3}$ French crowns *per* pound Sterling; I demand how many French crowns he ought to receive?

In placing the numbers, observe the 6th rule of the 10th chapter; which being done, the given numbers will stand as in the margin; and being reduced according to the rules of the 24th chapter, will stand thus,

l.		*crowns.*		*l.*
1	:	$3\frac{1}{3}$	: :	370

As $\frac{1}{1}$ is to $\frac{10}{3}$, so is $\frac{370}{1}$ to $1233\frac{1}{3}$.

So that I conclude he ought to receive $1233\frac{1}{3}$ French crowns at Paris for his 370 *l.* delivered at London.

Quest. 2. A merchant delivered at Amsterdam 587 l. Flemish, to receive the value thereof at Naples in ducats, the exchange $4\frac{4}{5}$ ducats *per* l. Flemish; I demand how many ducats he ought to receive?

The proportion is as followeth.

l. *Ducats.* *l.* *Ducats.*

As 1 is to $\frac{24}{5}$, so is $\frac{587}{1}$ to $2817\frac{3}{5}$.

So I find he ought to receive $2817\frac{3}{5}$ ducats at Naples for the 587 l. Flemish delivered at Amsterdam.

Quest. 3. A merchant at Florence delivereth 3478 ducatoons, to receive the value at London in pence, the exchange at $53\frac{1}{2}$ pence Sterling *per* ducatoon; I demand how much Sterling he ought to receive?

The proportion for resolution is,

Ducat. *d.* *Ducats.* *d.*

As $\frac{1}{1}$ is to $\frac{107}{2}$, so is $\frac{3478}{1}$ to 186073.

which is equal to 775 l. 6 s. 1 d. for the answer.

I

I might here (according to the custom of arithmetical writers) lay down tables for the reduction of foreign coins into English; but by reason of their instability (for they continue not at a constant standard, as our Sterling money doth, but are sometimes raised and sometimes depressed) I shall forbear.

4. When there is a comparison made between more than two different coins, weights, or measures, there arise ordinarily two different cases from such a comparison.

1. When it is required to know how many pieces of the first coin, weight, or measure, are equal in value to a known number of pieces of the last coin, weight, or measure.

2. When it is required to find out how many pieces of the last coin, weight, or measure, are equal in value to a given number of the first sort of coin, weight, or measure.

An example of the first case may be this, viz.

Quest. 4. If 150 pence at London are equal to 3 ducats at Naples, and $4\frac{4}{5}$ ducats at Naples make $34\frac{1}{2}$ shillings at Brussels; then how many pence at London are equal to 138 shillings at Brussels? *Facit* 960 d.

This question may be resolved by two single rules of three: for, first, I say,

If $\frac{3}{1}$ ducats at Naples make 150 d. at London, how many pence will $4\frac{4}{5}$ ducats make? *Ans.* 240 d.

By the foregoing proportion, we have discovered that $4\frac{4}{5}$ ducats at Naples make 240 pence at London: and by the tenour of the question, we see that $4\frac{4}{5}$ ducats at Venice make $34\frac{1}{2}$ shillings at Brussels; therefore 240 d. at London are equal to $34\frac{1}{2}$ shillings at Brussels, (for the things that are equal to one and the same thing, are also equal to one another); wherefore we have a way laid open to give a solution to this question by another single rule of three, whose proportion is,

As $34\frac{1}{2}$ shillings at Brussels is to 240 pence at London, so is 138 shillings at Brussels to 960 pence at London; which is the answer to the question.

An example of the second case may be this, viz.

Quest. 5. If 40 lb. Advoirdupois weight at London is

equal to 36 lb. weight at Amsterdam, and 90 lb. at Amsterdam makes 116 lb. at Dantzic; then how many pounds at Dantzic are equal to 112 lb. Advoirdupois weight at London? *Ans*. $129\frac{23}{25}$ lb. at Dantzic.

This question is likewise answered by two single rules of three, *viz*. First I say,

As 36 lb. at Amsterdam is to 40 lb. at London,
So is 90 lb. at Amsterdam to 100 lb. at London.

And by the question you find, that 90 lb. at Amsterdam is 116 lb. at Dantzic; and therefore 100 lb. at London is likewise equal thereunto; wherefore again I say,

As 100 lb. at London is to 116 lb. at Dantzic,
So is 112 lb. at London to $129\frac{23}{25}$ lb. at Dantzic.

By which I find that $129\frac{23}{25}$ lb. at Dantzic are equal to 112 lb. Advoirdupois weight at London.

5. There is a more speedy way to resolve such questions as are contained under the two cases before mentioned, laid down by Mr Kersey in the third chapter of his appendix to Wingate's arithmetic, where he hath given two rules for the resolution of the questions pertinent to the two said cases.

6. But I shall lay down a general rule for the solution of both cases. And first, Let the learner observe the following directions in placing of the given terms, *viz*.

7. Let there be made two columns, and in these columns so place the given terms, one over the other, as that in the same column there may not be found two terms of the same kind, one with the other.

Having thus placed the terms, the general rule is,

Observe which of the said columns hath the most terms placed in it, and multiply all the terms therein continually, and place the last product for a dividend; then multiply the terms in the other column continually, and let the last product be a divisor; then divide the said dividend by the said divisor, and the quotient thence arising is the answer to the question.

So the example of the first of the said cases being again repeated, *viz*. If 150 pence at London make three ducats at Naples, and $4\frac{4}{5}$ ducats at Naples make $34\frac{1}{2}$ shillings at Brussels; then how many pence at London are equal to 138 shillings at Brussels?

The

The terms being placed according to the 7th rule, will stand as followeth.

	A	B	
Pence at London	150	3	Ducats at Naples.
Ducats at Naples	$4\frac{4}{5}$	$34\frac{1}{2}$	Shillings at Bruffels.
Shillings at Bruffels	138		

Having thus placed the terms, that in neither column there are two terms of one kind, then obferve, that the column under A hath moft terms in it; therefore they muft be multiplied together for a dividend, *viz.* 150 multiplied by $4\frac{4}{5}$, produceth $\frac{3600}{5}$; which multiplied by 138, produceth $\frac{496800}{5}$ for a dividend; then in the column under B there are 3, and $34\frac{1}{2}$, which multiplied together, produce $\frac{207}{2}$ for a divifor; then having divided $\frac{496800}{5}$ by $\frac{207}{2}$, the quotient is 960 pence for the anfwer to the queftion.

Again, Let the example of the fecond cafe be again repeated, *viz.* If 40 lb. Avoirdupois weight at London make 36 lb. weight at Amfterdam, and 90 lb. at Amfterdam make 116 lb. at Dantzic; then how many pounds at Dantzic are equal to 112 lb. Avoirdupois weight at London?

The terms being difpofed according to the 7th rule foregoing, will ftand thus:

	A	B	
lb. at London	40	36	lb. at Amfterdam.
lb. at Amfterdam	90	116	lb. at Dantzic
		112	lb. at London.

Whereby I find that the terms under B multiplied together, produce 467712 for a dividend; and the terms under A, *viz.* 40 and 90, produce 3600 for a divifor; and divifion being finifhed, the quotient giveth $129\frac{3312}{3600}$ or $129\frac{23}{25}$ pounds Dantzic for the anfwer.

C H A P. XXXI.

Single Pofition.

1. NEgative arithmetic, called *the rule of falfe*, is that by which we find out a truth, by numbers invented or fuppofed. And this is either fingle or double.

2. The rule of fingle pofition, is, when at once, *viz.* by

by one false position, or feigned number, we find out the true number sought.

3. In the single rule of false, when you have made choice of your position, work it according to the tenour of the question, as if it were the true number sought; and if by the ordering your position you find either the result too much or too little, you may then find out the number sought by the proportion following, *viz.*

As the result of your position is to the position, so is the given number to the number sought. For example:

Quest. 1. A person having about him a certain number of crowns, said, If a fourth, and third, and sixth of them were added together, they would make just 45 crowns; now I demand the number of crowns he had about him? *Ans.* 60 crowns.

To resolve this question, I suppose he had 24 crowns, (or any other number that will admit of the like division); now the fourth of 24 is 6, and the third is 8, and the sixth is 4; all which parts (*viz.* 6, 8, and 4) being added together, make but 18, but it should be 45. Wherefore I say, by the rule of three,

As 18, the sum of the parts, is to the position 24, so is 45, the given number, to 60, the true number sought.

For the fourth of 60 is 15, and the third of 60 is 20, and the 6th of 60 is 10; which added together make 45.

Quest. 2. Three persons, *viz.* A, B, C, thus discourse together concerning their age. Quoth B to A, I am as old, and half as old again as you; then quoth C to B, I am twice as old as you; then quoth A to them, and I am sure the sum of all our ages is 165. Now I demand each man's age? *Ans.* A 30, B 45, C 90 years of age; which added together, make 165.

C H A P. XXXII.

Double Position.

1. THE rule of double position is, when two false positions are assumed to give a resolution to the question propounded.

2. When any question is stated in double position, make such a cross as in the margin. *a* X *b* / *d* X *c*

3. Then

3. Then make choice of any number you think may be convenient for your working, which call your first position, and place it at the end of the cross at *a*; then work with this position as if it were the true number sought, according to the nature of your question; then having found out your error, either too much or too little, place it on the side of the cross at *d*; then make choice of another number of the same denomination with the first position, (which call your second position), and place it on the other side of the cross at *b*; then work with this position as with the former; and having found out your error, either too much or too little, place it on the side of the cross at *c*, and then the positions will stand at the top of the cross, and the errors at the bottom, each under its correspondent position; and then multiply the errors into the positions crosswise; that is to say, multiply the first position by the second error, and the second position by the first error, and put each product over its position.

4. Having proceeded so far, then consider whether the errors are both alike; that is, whether they are both too much or both too little; and if they are alike, then subtract the lesser product from the greater, and set the remainder for a dividend; then subtract the lesser error from the greater, and let the remainder be a divisor; then the quotient arising by this division is the answer to the question.

5. But if the errors are unlike, that is, one too much, and the other too little, then add the products of the positions and errors together, and their sum shall be a dividend; then add the errors together, and their sum shall be a divisor; and the quotient arising thence, is the answer. Which two last rules may be kept in memory by the verse following, *viz.*

When errors are of unlike kinds,
Addition doth ensue;
But if alike, subtraction finds
Dividing work for you.

Quest. 1. A, B, and C, built a house which cost 76 l. of which A paid a certain sum unknown, B paid as much as A, and 10 l. over, and C paid as much as A and B; now I desire to know each man's share in that charge?

Having

Having made a cross according to the second rule, I come, according to the third rule, to make choice of my first position. And here I suppose A paid 6 l. which I put upon the cross,

I.				*I.*
A 6				A 9
B 16				B 19
C 22	120	168	288	C 28
Sum 44	6	X	9	Sum 56
76	12)		(14	76
44	32		20	56
		12		
Error 32				Error 20

as you see; then B paid 16 l. for it is said he had paid 10 l. more than A; and C paid 22 l. for it is said he paid as much as A and B: then I add their parts, and they amount to 44. But it is said, they paid 76 l. wherefore there is 32 too little, which I note down at the bottom of the cross under its position, for the first error.

Secondly, I suppose A paid 9 l. then B paid 19 l. and C 28 l. all which added together, makes 56. But they should make 76; wherefore the error of this position is 20; which I put at the bottom of the cross under its position, for the second error. Then I multiply the errors and positions crosswise, *viz.* 32 (the error of the first position) by 9 (the second position), and the product is 288; then I multiply 20 (the error of the second position) by 6 (the first position), and the product is 120; which products I place on the top of the cross, over their respective positions.

Then (according to the 4th rule) I subtract the lesser product from the greater, *viz.* 120 from 288, because the errors are both alike, (*viz.* too little), and there remaineth 168 for a dividend; then I subtract 20 (the lesser error) from 32 (the greater error), and the remainder is 12 for a divisor; then I divide 168 by 12, and the quotient is 14 for the answer; which is the share of A in the payment.

6. Again, secondly, if the errors had been both too big, it had had the same effect; as appeareth by the following work. For, first, I suppose A paid 20 l. then B paid 30 l. and C 50 l. which in all is 100 l. But it should have been no more than 76; wherefore the first error is 24 too much. Again, I suppose A paid 18; then B must pay 28 l. and C must pay 46 l. which in all is 92 l. But it should have been but 76 l.; wherefore the second error is 16 too much.

much. Then I multiply 20 (the firſt poſition) by 16 (the ſecond error), and the product is 320. Again I multiply 18 (the ſecond poſition) by 24 (the firſt error), and the product is 432. Then, becauſe the errors are both too much, I ſubtract 320 (the leſſer product) from 432 (the greater product), and there remaineth 112 for a dividend; likewiſe I ſubtract 16 (the leſſer error) from 24 (the greater error), and the difference is 8 for a diviſor; then perform diviſion, and the quotient is 14, as before, for the anſwer.

	l.					l.
A	20				A	18
B	30				B	28
C	50	320	112	432	C	46
Sum	100	20	X	18	Sum	92
Subtr.	76	8)		(14	Subtr.	76
Error	24	24		16	Error	16
			8			

Again, thirdly, If the errors had been, the one too big, and the other too little, reſpect being had to the 5th rule foregoing, the anſwer would have been the ſame. As, thus, I take for my firſt poſition 6, and then the error is 32 too little; then I take for my ſecond poſition 18, and then the error is 16 too much; then I multiply the poſitions and errors croſſwiſe, and the products are 96 and 576; and becauſe the errors are unlike, *viz.* one too big, and another too little, I add the products 96 and 576 together, and their ſum is 672 for a dividend: I likewiſe add the errors 32 and 16 together, and their ſum is 48 for a diviſor; then having finiſhed diviſion I find the quotient to be 14; which is the anſwer, as was found out at the two ſeveral trials before.

96	672	576
6	X	18
48)		(14
32		16
	48	

For proof of the work, I ſay,	—	l.
If A paid — — — — — —		14
Then B paid 14 and 10 (that is)	—	24
Then C paid 14 and 24 (that is)	—	38
The ſum of all is — — —		76

which is the total value of the building, and equal to the given number.

Thoſe

Those who desire to see the demonstration of this rule, let them read the 7th chapter of Mr Kersey's *Appendix to Mr Wingate's arithmetic*, Petiscus in the 5th book of his *Trigonometria*, or Mr Oughtred in his *Clavis mathematica*.

Quest. 2. Three persons, A, B, and C, thus discoursed together concerning their age. Quoth A, I am 18 years of age; quoth B, I am as old as A and $\frac{1}{2}$ C; and quoth C, I am as old as you both, if your years were added together. Now I desire to know the age of each person? *Ans.* A is 18, B is 54, and C is 72 years of age.

Quest. 3. A father lying at the point of death, left to his three sons, *viz.* A, B, C, all his estate in money, and divided it as followeth, *viz.* To A he gave the half wanting 44 l. to B he gave a third and 14 l. over, and to C he gave the remainder, which was 82 l. less than the share of B. Now I demand what was the sum left, and each man's part? *Ans.* The sum bequeathed was 588 l. whereof A had 250 l. B had 210 l. and C had 128 l.

Quest. 4. Two persons, *viz.* A and B, had each in their hands a certain number of crowns; and A said to B, If you give me one of your crowns, I shall have five times as many as you; and said B to him again, If you give me one of yours, then we shall each of us have an equal number. Now I demand how many crowns each person had? *Ans.* A had 4, and B had 2 crowns.

Quest. 5. What number is that unto which if I add one fourth of itself, and from the sum subtract one eighth of itself, the remainder will be 216? *Ans.* 192.

Many more questions may be added; but these well understood, will be sufficient (even to the meanest capacity) for the resolution of any other question pertinent to this rule.

There may be an objection made, because we have not treated particularly upon interest and rebate; but the operation of such questions being more applicable to decimals, they are omitted, till we come to acquaint the learner with decimal arithmetic.

Laus Deo soli.

FINIS.

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